A PRODUCTION MODEL FOR A FLEXIBLE PRODUCTION SYSTEM AND PRODUCTS WITH SHORT SELLING SEASON

MOUTAZ KHOUJA AND ABRAHAM MEHREZ

We address a practical problem faced by many firms. The problem is deciding on the production levels for a product that has a very short selling season. The firm has a full period to produce and meet a lumpy demand which occurs at the end of the period. The product is no longer demanded after the end of the period. A constant production rate which minimizes average unit cost may increase holding costs. Varying the production rate at discrete points in time may increase production costs but may also decrease holding costs. In addition, allowing changes in the production rate enables the incorporation of forecast revisions into the production plan. Therefore, the best production plan depends on the flexibility of the production system and on the holding cost. In this paper, we formulate and solve a model of this production planning problem. Two models are developed to deal with two types of the average unit cost function. Numerical examples are used to illustrate the results of the model.

1. Introduction

Consider the problem faced by a firm which produces a product for a full period to meet a demand which is concentrated at the end of that period. There is no demand for the product after the end of the period. This problem is common for many suppliers of products treated in the single period model [8]. However, unlike retailers, suppliers are not faced with the problem of discounting the product if any inventory remains at the end of the period [15], because they produce to the orders of the retailers. The problem for the suppliers is deciding on the production levels of products. A constant production rate which minimizes average unit cost may increase holding costs. Varying the production rate at discrete points in time may increase production costs but may also decrease holding costs. Therefore, the best production plan depends on the flexibility of the production system and on the holding cost. In addition, allowing changes in the production rate enables the incorporation of forecast revisions into the production plan.

The cost of producing at different production rates depends on the volume flexibility of the production system. Volume flexibility is defined as the ability of a system to operate profitably at different output levels [13]. The cost of producing at different production

rates has two components. The first component is the cost of switching from one production rate to another, and the second is the penalty cost of deviating from the production rate which minimizes average unit cost. The switching cost arises from the need to change the size of the work force by hiring and laying off workers [6]. As for the penalty cost, stability of production costs over varying production volumes has been suggested as a measure of volume flexibility [3]. Ramasesh and Jayakumar [11] used the shape of the average unit cost curve to measure volume flexibility. Flat average unit cost curve indicates a volume flexible production system. Average unit cost initially decreases because fixed costs are spread over more units. At higher volumes, increases in tool wearout [2, 12] and increases in error rates cause average unit cost to increase. The treatment of production rates as decision variables has been incorporated in many inventory models [4, 7, 10, 14].

Several authors have developed production planning models which allow the incorporation of forecast revisions into the production plan [1, 9]. Bitran et al. [1] dealt with a system that produces several families of style goods. A family is defined as a set of items consuming the same amount of resources and sharing the same setup. Bitran et al. assumed that the setup cost associated with changeover from one family to the next is large enough that managers attempt to produce each family once in the planning horizon. Also, the authors assumed that the mean demand for each family is invariant over the horizon whereas item demands are forecasted in each period. Demand occurs in the last season of the horizon and demand estimates for items are revised every period. The problem is finding item production quantities which will maximize the expected profit. Bitran et al. assumed the demand of items in a family follow a joint normal distribution and that each period has limited production capacity. The problem was formulated as a stochastic mixed integer programming problem and by exploiting its hierarchical structure (families and then items), the authors formulated and solved a deterministic mixed integer programming problem which provided an approximate solution. Matsuo [9] observed that a limitation of Bitran et al's model [1] is that it included discrete production periods and each family is assigned to exactly one period which works well only if the number of families is much larger than the number of periods. Also, the complexity of Bitran et al's method made sensitivity analysis difficult. To avoid the limitations of Bitran et al's model, Matsuo used a continuous treatment of time and formulated the problem as a two-stage stochastic sequencing model. In stage I, a sequence of production quantities of families is determined at the beginning of planning horizon. In stage II, the production quantities of items in each family are determined using the revised demand forecast. Matsuo [9] developed and tested a heuristic procedure for solving the problem. Both of the above models dealt with constant production rate and constant unit production cost.

In this paper, we formulate and solve two production planning models. In the first model, the firm incurs a linear penalty cost for deviating from the minimum average unit cost production rate whereas in the second model it incurs a quadratic penalty cost. In Section 2, we introduce the basic model in which a single production rate change is allowed during the period and solve the linear and then the quadratic penalty case. In Section 3, we allow revisions to the demand forecast and multiple production rates for the linear penalty case. We close with a discussion and suggestion for future research in Section 4.
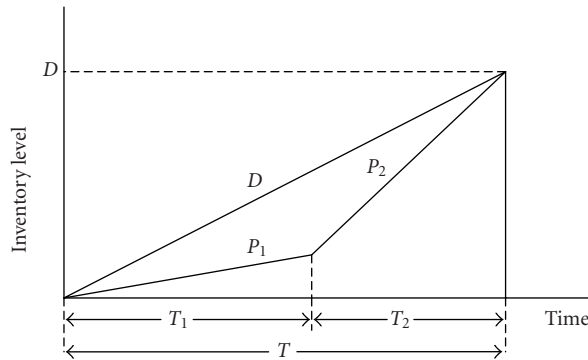
Figure 2.1. Two production plans to meet lumpy demand.

2. Model I: single production rate change

Consider a firm which has a very short selling season relative to the total production period. Management must design a production plan to meet the forecasted lumpy demand at the end of the period. Demand for the product exists only in the current period. A production plan specifies production rates during the period. We initially restrict the number of possible production rates to two, which implies at most one production rate change.

Figure 2.1 shows two possible production plans. The first plan is to produce at a constant rate of D for the total duration of the period T. The second plan is to produce at a rate of P_1 during T_1 and then increase the production rate to P_2 during T_2. The two plans incur different holding and production costs. The problem is analyzed under the following assumptions:

(1) demand is concentrated at the end of the period,
(2) holding cost is linear in the number of units held,
(3) the cost of changing production rates increases linearly in the difference between the rates,
(4) average unit cost as a function of the output rate is convex with a minimum at P_0.

Assumption 2 is commonly used in inventory management. The cost of changing production rates in assumption 3 has been justified in the production planning literature [6]. The convexity of the average unit cost in assumption 4 is well accepted in the production literature [11].

Define the following notation:

T_1 = duration with a production rate of P_1,
P_1 = production rate during the first part of the cycle (T_1),
T_2 = duration with a production rate of P_2,
P_2 = production rate during the second part of the cycle (T_2),
C_0 = the minimum average unit cost (i.e., the average unit cost at the design volume),
P_0 = the design volume of the production system (i.e., the volume which minimizes average unit cost),
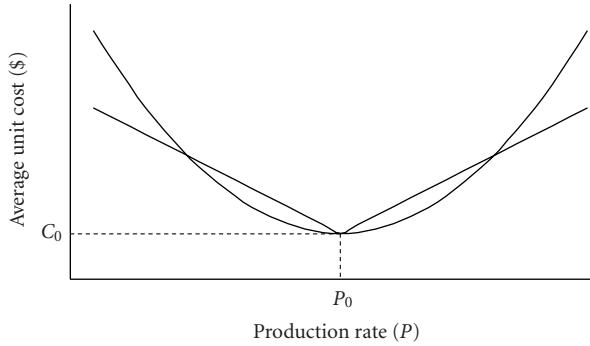
Figure 2.2. Two cases of the average unit cost.

R = annual holding cost as a percentage of inventory value,
C_1 = average unit cost at production rate P_1,
C_2 = average unit cost at production rate P_2, and
$f(P_1, P_2)$ = the cost of switching from a production rate of P_1 to P_2,
D = demand at the end of the period.

If the average unit cost is symmetric around the minimum, then optimal solutions must satisfy $P_1 \leq P_0 \leq P_2$. Otherwise, reducing P_1 to P_0 and increasing P_2 to P_0 will reduce the holding cost, production cost, and production rate switching cost. We assume that the demand is a random variable with a minimum D_L, a mean D_μ, and maximum D_M and that the capacity of the production system is sufficient to produce at the minimum average unit cost for the maximum demand, $D_M \leq P_0$.

We assume C_1 is a convex decreasing function in P_1 and C_2 is a convex increasing function in P_2. Linear and quadratic symmetrical cases of C_1 and C_2 are shown in Figure 2.2. We also assume $f(P_1, P_2)$ depends only on the absolute difference $|P_1 - P_2|$. The total annual cost is

$$TC = \frac{1}{2}RC_1 T_1^2 P_1 + RC_1 T_1 T_2 P_1 + \frac{1}{2}RC_2 T_2^2 P_2 + C_1 T_1 P_1 + C_2 T_2 P_2 + f(P_1, P_2), \quad (2.1)$$

where

$$T_1 + T_2 = T, \quad (2.2)$$

$$P_1 T_1 + P_2 T_2 = D. \quad (2.3)$$

Without loss of generality, we assume $T = 1$. Equation (2.2) becomes

$$T_1 + T_2 = 1, \quad (2.4)$$

Lemma 2.1 shows that to minimize total cost, the production rate must be reduced or kept at D in the first part of the period to avoid large holding cost. A reduction is likely to be optimal if the holding cost is high and the system is volume flexible, which implies that deviations from the minimum cost production rate do not incur large penalty.

The next two sections provide solutions for the linear and quadratic penalty cases. Let the superscript * denote optimality.

LEMMA 2.1. *The optimal solution to minimizing TC satisfies $P_1^* \leq D$ and $P_2^* \geq D$.*

Proof. See appendix.   □

2.1. The linear penalty cost case. In this case, deviations from the minimum average unit cost production rate results in the same linear penalty in either direction as shown in Figure 2.2. The average unit cost during T_1 and T_2 are

$$C_1 = C_0 + a|P_1 - P_0|, \tag{2.5}$$

$$C_2 = C_0 + a|P_2 - P_0|, \tag{2.6}$$

respectively. Also, the switching cost is linear and is given by

$$f(P_1, P_2) = K|P_1 - P_2|, \tag{2.7}$$

where K is the penalty cost for changing the production rate by one unit.

Equations (2.4)–(2.7) are used to eliminate C_1, C_2, P_2, T_2, and $f(P_1, P_2)$ from TC. Also, since we focus on systems for which $P_2^* \geq P_0$ or $P_2^* = D$, we eliminate the absolute value operator. Simplifying TC gives:

$$
\begin{aligned}
TC = \big[& aD(4P_1 - 2P_0 + DR - P_0R) + 2P_1(K - a) \\
& - 2[K + a(D - P_1)](D - P_1)/(T_1 - 1) \\
& + a[P_0(4P_1 + DR + 3P_1R) - 2P_1(2P_1 + (D + P_1)R)]T_1 \\
& - 2a(P_0 - P_1)P_1RT_1^2 + C_0[P_1RT_1 + D(2 + R - RT_1)]\big]/2.
\end{aligned}
\tag{2.8}
$$

Define

$$T_{1C} = \frac{1 + R - \sqrt{1 + R}}{R}. \tag{2.9}$$

LEMMA 2.2. *TC is concave in P_1 for $T_1 \in (0, T_{1C})$ and convex for $T_1 \in (T_{1C}, 1]$.*

Proof. See appendix.   □

For any $T_1 \in [0, T_{1C})$, by Lemmas 2.1 and 2.2, the optimal P_1 is $P_1 = 0$ or $P_1 = D$. The later case is the constant production rate solution. Substituting $P_1 = 0$ into (2.9) and differentiating TC twice with respect to T_1 gives

$$\frac{\partial^2 TC}{\partial T_1^2} = \frac{2D(K + aD)}{(1 - T_1)^3} > 0. \tag{2.10}$$

Thus, TC is convex in T_1 for $P_1 = 0$. The necessary condition for T_1 to be optimal is obtained from setting $\partial TC/\partial T_1 = 0$, which gives:

$$T_{1r} = 1 - \sqrt{\frac{2(K + aD)}{R(C_0 - aP_0)}}. \tag{2.11}$$

Obviously, the optimal T_{1r} must satisfy $0 < T_{1r} \le 1$. By inspection, T_{1r} in (2.11) satisfies $T_{1r} \le 1$. If $T_{1r} > 0$ then $P_1 = 0$ and $T_1 = T_{1r}$ are local minima and a comparison between $TC(P_1 = 0)$ and $TC(P_1 = D)$ is needed. If $T_{1r} < 0$ or $T_{1r} = 0$ then $P_1 = P_2 = D$ is optimal.

An examination of the expression of T_{1r} provides some useful insights into the model:

(1) T_{1r} is increasing in R which implies that for large holding cost, it is optimal to delay production for a long duration of time to avoid the large cost of holding inventory. Following the same argument, for small holding cost, $T_{1r} < 0$ and it is optimal to start producing immediately.

(2) For large cost of switching the production level by one unit, K, $T_{1r} < 0$ and it is optimal not to have a change in the production rate from the previous period or during the period.

(3) The smaller the value of the penalty parameter a (which implies a more flexible system), the longer the optimal T_{1r}. Since the penalty of deviating from the minimum average unit cost production rate is small, it is beneficial to reduce the holding cost by producing at a large production rate at the end of the period. Under extreme conditions when $a \approx 0$ and $K \approx 0$ (the system is completely flexible), $T_{1r} \approx 1$ and all the production is concentrated at the end of the period.

Example 2.3. Consider a firm with the following data: $P_0 = 110\,000$ units/year, $D = 100\,000$ units/year, $C_0 = \$50$, $R = 0.15$, $K = 0.05$, and $a = 0.00001$. If the firm follows a constant production rate strategy of $P_1 = P_2 = D$ then $TC = \$5\,371\,328$ per year. Using the proposed model gives $P_1^* = 0$, $T_1^* = 0.4649$ years, $P_2^* = 186\,892$ units/year, $T_2^* = 0.5351$ years, and $TC = \$5\,289\,973$ per year which represents a saving of $\$81\,355$ per year over the constant production rate plan. For a higher holding cost of $R = 0.30$, the model yields $P_1^* = 0$, $T_1^* = 0.6216$ years, $P_2^* = 264\,305$ units/year, $T_2^* = 0.3784$ years, and $TC = \$5\,460\,040$ per year which represents a saving of $\$285\,799$ per year over the constant production rate strategy which has a $TC = \$5\,745\,839$ per year. In cases where the holding cost is low and/or the switching cost is high, the constant production rate strategy is optimal. For example, if $R = 0.10$ and $K = \$2$ then $P_1^* = P_2^* = D$.

2.2. The quadratic penalty cost case. In this case, deviations from the design production rate yield symmetric quadratic penalty in either direction (see Figure 2.2). The average unit cost during T_1 and T_2 are

$$C_1 = C_0 + a(P_1 - P_0)^2, \tag{2.12}$$

$$C_2 = C_0 + a(P_2 - P_0)^2, \tag{2.13}$$

respectively. Equations (2.4), (2.5), (2.8), (2.12), and (2.13) are used to eliminate C_1, C_2, P_2, T_2, and $f(P_1,P_2)$ from TC and then setting $\partial TC/\partial P_1 = 0$ gives the necessary conditions for P_1 to be optimal as

$$P_1 = \frac{-x \pm \sqrt{x^2 - 4yW}}{2y},$$

(2.14)

where

$$x = 2aT_1(4P_0(1 + R(1 - T_1))(T_1 - 1) + 3DT_1(2 + R(1 - T_1))),$$

(2.15)

$$y = 3aT_1(2 - 4T_1 - R(1 - T_1)(3T_1 - 2)),$$

(2.16)

$$W = a\left(-3D^2(2 + R(1 - T_1)) - 4DP_0(2 + R(1 - T_1))(1 - T_1) + RP_0^2(T_1 - 1)^2\right)$$
$$+ (2K + C_0R(T_1 - 1))(T_1 - 1)T_1.$$

(2.17)

Our analysis shows that $(-x + \sqrt{x^2 - 4yW})/2y$ is the positive root. Therefore, $P_{1,1} = (-x + \sqrt{x^2 - 4yW})/2y$ is substituted into (2.1) and a one-dimensional search is conducted to find T_1^*. Since no proof of convexity in T_1 can be obtained for this case, we can only claim a local minimum. T_1^* is then substituted in (2.14) and by Lemma 2.1 if $P_{1,1} > D$, then $P_1^* = D$, whereas if $P_{1,1} < 0$, then $P_1^* = 0$. If $P_1^* \neq D$ then the total cost of producing at a constant production rate $TC(P_1 = P_2 = D)$ must be compared to the local minimum of $TC(P_1^*, T_1^*)$.

Example 2.4. We use the same data as in Example 2.3 with a quadratic average unit cost function for which $a = 2 \times 10^{-9}$. If the firm follows a constant production rate strategy of $P_1 = P_2 = D$, then $TC = \$5\,401\,500$ per year. Using the proposed model gives $P_1^* = 0$, $T_1^* = 0.14085$ years, $P_2^* = 116\,394$ units/year, $T_2^* = 0.85915$ years, and $TC = \$5\,336\,700$ per year, which represents a saving of $\$64\,800$ per year. For a less flexible system with $K = \$10$, the constant production rate strategy $P_1 = P_2 = 100\,000$ with a cost of $TC = \$6\,396\,500$ is optimal.

3. Model II: forecast revisions and multiple production rates

We now treat demand as a random variable whose probability density function can be forecasted by management. Obviously, the forecasts improve as the selling season draws closer and production decisions must be revised accordingly. We analyze only the linear penalty case. Let

D_N = the largest demand for which $P_1 = 0$ is optimal.

Because of forecast revisions, we no longer set $T = 1$. For any T, (2.11) becomes

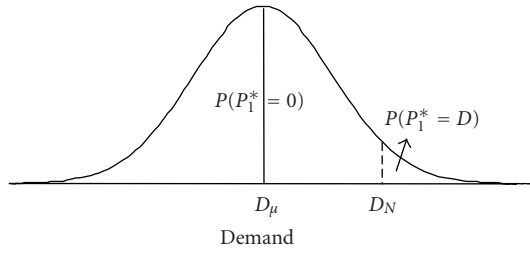$$T_{1r} = T - \sqrt{\frac{2(K + aD)}{R(C_0 - aP_0)}}.$$

(3.1)

Figure 3.1. Probability density function of demand.

From (3.1), D_N is given by setting $T_{1r} = 0$ (otherwise $T_1^* < 0$) which gives

$$D_N = \frac{-2K + (C_0 - aP_0)RT^2}{2a}.$$ (3.2)

For any value of demand satisfying $D < D_N$, $P_1^* = 0$, and for any value satisfying, $D > D_N$, $P_1^* = D$ (see Lemma 2.1). Therefore, the probabilities of the different values of P_1^* being optimal are as shown in Figure 3.1. Let I_i be the beginning inventory at time of forecast revision i. The following procedure is designed to optimize the production plan at each forecast revision.

Production planning procedure
  Step 0. Set $T = 1$, $i = 0$, and $I_0 = 0$.
  Step 1. Forecast the demand distribution.
          Set $D_L = D_L - I_i$, $D_M = D_M - I_i$, and $D_\mu = D_\mu - I_i$.
  Step 2. Compute D_N using (3.2).
  Step 3. Compute $P(P_1^* = 0) = P(D < D_N)$ and $P(P_1^* = D) = 1 - P(D < D_N)$.
  Step 4. If $P(P_1^* = 0) > P(P_1^* = D)$ then set $P_1^* = 0$, find T_1^* using (3.1), and find T_2^* and P_2^* using (2.2) and (2.3), respectively.
          If $P(P_1^* = 0) < P(P_1^* = D)$ and then set $P_2^* = D_\mu/T$.
  Step 5. If no more forecast updates are to be made, execute the plan.
          If more forecast updates are to be made at time T_{i+1}, then
          Set $i = i + 1$, $T = T - T_i$, $I_i = I_{i-1} + \max\{(T_i - T_{i-1} - T_1^*), 0\}P_2^*$ and go to Step 1.

Example 3.1. Consider a problem with $a = 0.000022$, $K = 0.1$, and the rest of the parameters as given in Example 2.3. Suppose the firm starts at time zero (i.e., the beginning of the period) with a forecast of a uniformly distributed demand with the parameters shown in Table 3.1. This forecast is updated at four points in time as shown in Table 3.1. Using (3.2) yields $D_N = 158\,000$ which results in $P(P_1^* = 0) = 1$ and $T_1^* = 0.2225$ but since the forecast is revised at 0.2, $T_1^* = 0.20$. After 0.2 periods, $D_N = 99\,250$ which results in $P(P_1^* = 0) = 0.5625$ and $T_1^* = 0.00489$. Therefore, production starts at $0.2 + 0.00489 = 0.20489$ at $P_2^* = 123\,245$ units/period. The forecast is updated at 0.4, 0.6, and 0.8 periods and new production rates are established according to the proposed procedure as shown in Table 3.1.

Table 3.1. Production plan with 4 forecast revisions.

i	T_i	T	D_L	D_M	D_μ	I_i	$D_L - I_i$	$D_M - I_i$	$D_\mu - I_i$	D_N	T_1^*	P_1^*	$P(P_1^* = 0)$	T_2^*	P_2^*
0	0	1	80000	107000	93500	0	80000	107000	93500	158000	0.2225	0	1	0.7775	120262
1	0.2	0.8	88000	108000	98000	0	88000	108000	98000	99250	0.00489	0	0.5625	0.7951	123254
2	0.4	0.6	87000	106000	96500	24048	62952	81952	72452	53848	—	120753	0	0.4000	120753
3	0.6	0.4	90000	105000	97500	48199	41801	56801	49301	21407	—	123253	0	0.2000	123253
4	0.8	0.2	97000	97000	97000	72849	24151	24151	24151		—	120753	0	0.0000	120753
	1	0				97000									

Table 3.2. A reactive production plan to forecast revisions.

Time		Production rate	Unit cost	Production cost	Holding cost for current units	Holding cost for previous production	Production rate change cost
From	To	(units/period)	($/unit)	($)	($)	($)	($)
0.000	0.200	93500	50.36	941788	14127	113015	9350
0.200	0.400	99125	50.24	995993	14940	89639	563
0.400	0.600	96625	50.29	971936	14579	58316	250
0.600	0.800	99125	50.24	995993	14940	29880	250
0.800	1.000	96625	50.29	971936	14579	0	250
				4 877 647	73 165	290 850	10 663

Table 3.3. A production plan based on the proposed model.

Time		Production rate	Unit cost	Production cost	Holding cost for current units	Holding cost for previous production	Production rate change cost
From	To	(units/period)	($/unit)	($)	($)	($)	($)
0.000	0.205	0	52.42	0	0	0	0
0.205	0.400	123254	50.29	1209411	17698	0	12325
0.400	0.600	120753	50.24	1213247	18199	36282	250
0.600	0.800	123253	50.29	1239721	18596	72680	250
0.800	1.000	120753	50.24	1213247	18199	109871	250
				4 875 626	72 691	218 833	13 075

Tables 3.2 and 3.3 provide a comparison between a plan in which the production rate is set to meet the mean demand at a constant production rate at each forecast revision, and one which follows the suggested model, respectively. The implementation of the proposed model results in reducing the total cost from $5 252 324 to $5 180 226, a saving of 1.4%

4. Conclusions and suggestions for future research

This paper addresses the problem of deciding on the production levels for a product which has a very short selling season. The firm has a full period to produce and meet a lumpy demand which occurs at the end of the period. The best production plan depends on the flexibility of the production system and on the holding cost. Two models for dealing with linear and quadratic penalty functions for deviating from the minimum average unit cost production rate are developed. The models allow only a single production rate

Figure 4.1. Inventory level.

change. The model is extended to allow revisions to the demand forecast and multiple production rates for the linear penalty case.

Future research involve dealing with multi-item problem. In this case, the problem will also involve a scheduling aspect. For this scheduling aspect, to reduce holding cost, it may be best to produce items that are costly to hold and require short processing time at the end of the period and items with long processing time and low holding cost early in the period. A similar strategy was found to be optimal in the economic delivery and scheduling problem (ELDSP) [5].

Appendix

Proof of Lemma 2.1. Suppose $P_1 = P_x$, where $D < P_x \leq P_0$ during T_1, then $P_2 < D$ during T_2. Using Figure 4.1, the following observations are made with regard to the total cost components.

(1) Production costs. The same amount of production at a lower unit production cost can be achieved by setting $P_1 = 0$ during T_3 and $P_2 = P_x$ during $(1 - T_3)$ can be achieved.

(2) Holding costs. The cumulative inventory at any time is lower for $P_1 = 0$ during T_3 and $P_2 = P_x$ during $(1 - T_3)$ than $P_1 = P_x$. Also, the production cost of this inventory is lower. Therefore, the holding cost is lower for $P_1 = 0$ during T_3.

(3) Switching costs. The switching cost for the $P_1 = P_x$ plan is $K[P_x + (P_x - D)/T_2]$ whereas for the $P_1 = 0$ during T_3 plan it is KP_x. Therefore, the switching cost for the $P_1 = P_x$ plan is larger by $K(P_x - D)/T_2$.

From observations (1)–(3), the total cost for $P_1 = P_x$ is greater than the total cost for $P_1 = 0$ during T_3. Therefore, $P_1 = P_x > D$ cannot be optimal. $\square$

Proof of Lemma 2.2. The first derivative of TC with respect to P_1

$$\frac{\partial TC}{\partial P_1} = \frac{K + 2a(D - P_1)}{(T_1 - 1)} + a\left[2D - 2P_1 - (P_0 - 2P_1)RT_1^2\right]$$
$$+ \left[C_0R + a[P_0(4 + 3R) - 2(DR + 2P_1(2 + R))]R_1\right]/2$$

(4.1)

and the second derivative

$$\frac{\partial^2 TC}{\partial P_1^2} = \frac{2a\left(1 + R(T_1 - 1)^2 - 2T_1\right)T_1}{T_1 - 1}.$$

(4.2)

$\partial^2 TC/\partial P_1^2 = 0$ at $T_{11} = 0$, $T_{12} = (1 + R - \sqrt{1 + R})/R$, and $T_{13} = (1 + R + \sqrt{1 + R})/R$. Since $(1 + R - \sqrt{1 + R})/R > 0$ and $1 + (1 - \sqrt{1 + R})/R < 1$, $0 \leq T_{12} < 1$. Also, $T_{13} \geq 1$. For $T_{11} < T_1 < T_{12}$, $\partial^2 TC/\partial P_1^2 < 0$ and TC is concave, and for $T_{12} < T_1 < T_{13}$, $\partial^2 TC/\partial P_1^2 < 0$ and TC is convex. $\qquad\square$

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Moutaz Khouja: The University of North Carolina at Charlotte, Charlotte, NC 28223, USA
E-mail address: mjkhouja@email.uncc.edu

Abraham Mehrez: Ben-Gurion University of the Negev, Beer Sheva 84105, Israel

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Guest Editor

Donal O'Regan, Department of Mathematics, National University of Ireland, Galway, Ireland; donal.oregan@nuigalway.ie