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DECOMPOSITION SOLUTION FOR DUFFING AND VAN DER POL OSCILLATORS

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ABSTRACT. The decomposition method is applied to solve the Duffing and Van der Pol oscillators without customary restrictive assumptions [1-4] and without resort to perturbation methods.

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1. INTRODUCTION

The Duffing equation is written

$$\ddot{y} + \alpha\dot{y} + \beta y + \gamma y^3 = x(t) \tag{1.1}$$

The Van der Pol equation can be written

$$\ddot{y} + \alpha\dot{y} + \beta y + \gamma(d/dt)y^3 = x(t) \tag{1.2}$$

(If $\alpha = -\xi$, $\beta = 1$, $\gamma = \xi/3$, we have the form usually given.) Write $L = d^2/dt^2$, $R = \alpha(d/dt) + \beta$, $Ny = \gamma y^3$ in (1.1) and $\gamma(d/dt)y^3$ in (1.2) Thus both are written

$$Ly + Ry + Ny = x(t) \tag{1.3}$$

in the standard form for the decomposition method [1-3] where L^{-1} is the two-fold definite integral from 0 to t. Then,

$$Ly = x(t) - Ry - Ny. \tag{1.4}$$

Assuming initial conditions $y(0)$, $y'(0)$ are specified, let $y = \sum_{n=0}^{\infty} y_n$ and define y_0 by

$$y_0 = y(0) + ty'(0) + L^{-1}x(t). \tag{1.5}$$

Then

$$y_{n+1} = -L^{-1}\alpha(d/dt)y_n - L^{-1}\beta y_n - L^{-1}[Ny] \tag{1.6}$$

for $n \geq 0$.

2. SOLUTION OF THE PROBLEM

To get computable solutions, we need only substitute for Ny the sum $\gamma \sum_{n=0}^{\infty} A_n$ for the Duffing case and $\gamma(d/dt) \sum_{n=0}^{\infty} A_n$ for the Van der Pol case where the A_n are Adomian's polynomials [1-5] generated for the nonlinear term y^3 and representing it exactly in a rapidly converging series [1-5].

$$
\begin{aligned}
A_0 &= y_0^3 \\
A_1 &= 3y_0^2 y_1 \\
A_2 &= 3y_0^2 y_2 + 3y_0 y_1^2 \\
A_3 &= 3y_0^2 y_3 + 6y_0 y_1 y_2 + y_1^3 \\
A_4 &= 3y_0^2 y_4 + 3y_0 y_2^2 + 6y_0 y_1 y_3 + 3y_1^2 y_2 \\
A_5 &= 3y_0^2 y_5 + 6y_0 y_1 y_4 + 6y_0 y_2 y_3 + 3y_1^2 y_3 + 3y_1 y_2^2 \\
&\vdots
\end{aligned}
\tag{2.1}
$$

The deterministic problem is now solved since all components of y are determined. We use an n-term approximation $\phi_n = \sum_{i=0}^{n-1} y_i$ which, because of the rapid convergence, is generally sufficient with a very small n (say half a dozen or so terms) but easily carried as far as necessary since the integrals do not involve difficult Green's functions. Convergence has been previously established by Adomian [2,5] and has been shown [2] to be quite rapid.

For the stochastic case [2], none of the usual approximations of statistical linearization are necessary. The $x(t)$ need not be stationary nor Gaussian nor delta-correlated. Further α,β,γ and the initial conditions can be stochastic. No "smallness" assumptions are necessary for the stochastic processes and the nonlinearities. No linearization is used. We can allow $\alpha = \langle\alpha\rangle + \xi$, $\beta = \langle\beta\rangle + \eta$, $\gamma = \langle\gamma\rangle + \sigma$ and write $Ly = x - \langle\alpha\rangle(d/dt)y - \langle\beta\rangle y - \langle\gamma\rangle \sum_{n=0}^{\infty} A_n - \xi(d/dt)y - \eta y - \sigma \sum_{n=0}^{\infty} A_n$ and proceed as before with $y = \sum_{n=0}^{\infty} y_n$.

The result is a stochastic series from which statistics are obtained without the problems of statistical separability of quantities such as $\langle R y\rangle$ where $R = \xi d/dt - \eta$ which normally require closure approximations.

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