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INTUITIONISTIC FUZZY PROXIMITY SPACES

SEOK JONG LEE and EUN PYO LEE

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We introduce the concept of the intuitionistic fuzzy proximity as a generalization of fuzzy proximity, and investigate its properties. Also we investigate the relationship among intuitionistic fuzzy proximity and fuzzy proximity, and intuitionistic fuzzy topology.

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1. Introduction. As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [1]. Recently, Çoker and his colleagues [2, 3, 4] introduced the concept of intuitionistic fuzzy topology which is a generalization of fuzzy topology.

Katsaras [5, 6] introduced the concept of fuzzy proximity, and studied the relationship between fuzzy topology and fuzzy proximity. Liu [9] introduced the concept of L-fuzzy proximity for a lattice L, and Liu and Luo [10] studied the relation between L-fuzzy proximity and L-fuzzy uniformity. Also, Khare [7] studied the relationship between classical and fuzzy proximities.

In [8], we studied the relationship between fuzzy topology and intuitionistic fuzzy topology.

In this paper, we introduce the concept of the intuitionistic fuzzy proximity as a generalization of fuzzy proximity, and investigate its properties. Also we investigate the relationship among intuitionistic fuzzy proximity and fuzzy proximity, and intuitionistic fuzzy topology. Moreover, we find an adjunction between intuitionistic fuzzy proximity spaces and fuzzy proximity spaces.

2. Preliminaries. In this section, we recall some of the definitions and theorems related to fuzzy proximity and intuitionistic fuzzy topology.

Let X be a nonempty set and I the unit interval $[0,1]$. An *intuitionistic fuzzy set* A is an ordered pair

$$A = (\mu_A, \gamma_A), \tag{2.1}$$

where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership and the degree of nonmembership, respectively, and $\mu_A + \gamma_A \le 1$. Let $I(X)$ denote the set of all intuitionistic fuzzy sets in X.

Obviously every fuzzy set μ_A in X is an intuitionistic fuzzy set of the form $(\mu_A, 1-\mu_A)$.

Definition 2.1 [1]. Let $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ be intuitionistic fuzzy sets in X. Then

(1) $A \subseteq B$ if and only if $\mu_A \le \mu_B$ and $\gamma_A \ge \gamma_B$;
(2) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$;
(3) $A^c = (\gamma_A, \mu_A)$;
(4) $A \cap B = (\mu_A \wedge \mu_B, \gamma_A \vee \gamma_B)$;
(5) $A \cup B = (\mu_A \vee \mu_B, \gamma_A \wedge \gamma_B)$;
(6) $0_\sim = (\tilde{0}, \tilde{1})$ and $1_\sim = (\tilde{1}, \tilde{0})$.

Let f be a map from a set X to a set Y. Let $A = (\mu_A, \gamma_A)$ be an intuitionistic fuzzy set in X and $B = (\mu_B, \gamma_B)$ an intuitionistic fuzzy set in Y. Then $f^{-1}(B)$ is an intuitionistic fuzzy set in X defined by

$$f^{-1}(B) = (f^{-1}(\mu_B),\ f^{-1}(\gamma_B)) \tag{2.2}$$

and $f(A)$ is an intuitionistic fuzzy set in Y defined by

$$f(A) = (f(\mu_A),\ 1 - f(1 - \gamma_A)). \tag{2.3}$$

Definition 2.2 [3]. An *intuitionistic fuzzy topology* on X is a family $\mathcal{T}$ of intuitionistic fuzzy sets in X which satisfies the following properties:
(1) $0_\sim, 1_\sim \in \mathcal{T}$;
(2) if $A_1, A_2 \in \mathcal{T}$, then $A_1 \cap A_2 \in \mathcal{T}$;
(3) if $A_i \in \mathcal{T}$ for each i, then $\bigcup A_i \in \mathcal{T}$.
The pair $(X, \mathcal{T})$ is called an *intuitionistic fuzzy topological space*. Any element of $\mathcal{T}$ is called an *intuitionistic fuzzy open set* in X and the complement, an *intuitionistic fuzzy closed set*.

Definition 2.3 [2, 3]. Let $(X, \mathcal{T})$ be an intuitionistic fuzzy topological space and A an intuitionistic fuzzy set in X. Then the *fuzzy closure of* A is defined by

$$\mathrm{cl}(A) = \bigcap \{F \mid A \subseteq F,\ F^c \in \mathcal{T}\} \tag{2.4}$$

and the *fuzzy interior of* A is defined by

$$\mathrm{int}(A) = \bigcup \{G \mid A \supseteq G,\ G \in \mathcal{T}\}. \tag{2.5}$$

Theorem 2.4 [3]. *For any intuitionistic fuzzy set A in an intuitionistic fuzzy topological space $(X, \mathcal{T})$, there exist*
(1) $\mathrm{int}(A)^c = \mathrm{cl}(A^c)$;
(2) $\mathrm{cl}(A)^c = \mathrm{int}(A^c)$.

Theorem 2.5 [2]. *Let $(X, \mathcal{T})$ be an intuitionistic fuzzy topological space and $\mathrm{cl} : I(X) \to I(X)$ the fuzzy closure in $(X, \mathcal{T})$. Then for $A, B \in I(X)$, the following properties hold:*
(1) $\mathrm{cl}(0_\sim) = 0_\sim$;
(2) $A \subseteq \mathrm{cl}(A)$;
(3) $\mathrm{cl}(A \cup B) = \mathrm{cl}(A) \cup \mathrm{cl}(B)$;
(4) $\mathrm{cl}(\mathrm{cl}(A)) = \mathrm{cl}(A)$.

Definition 2.6 [3]**.** Let $\alpha,\beta \in [0,1]$ and $\alpha+\beta \le 1$. An *intuitionistic fuzzy point* $x_{(\alpha,\beta)}$ of X is an intuitionistic fuzzy set in X defined by

$$x_{(\alpha,\beta)}(y) = \begin{cases} (\alpha,\beta) & \text{if } y = x, \\ (0,1) & \text{if } y \ne x. \end{cases} \tag{2.6}$$

In this case, x is called the *support* of $x_{(\alpha,\beta)}$, α the *value* of $x_{(\alpha,\beta)}$ and β the *nonvalue* of $x_{(\alpha,\beta)}$. An intuitionistic fuzzy point $x_{(\alpha,\beta)}$ is said to *belong* to an intuitionistic fuzzy set $A = (\mu_A, \gamma_A)$ in X, denoted by $x_{(\alpha,\beta)} \in A$, if $\alpha \le \mu_A(x)$ and $\beta \ge \gamma_A(x)$.

Clearly an intuitionistic fuzzy point can be represented by an ordered pair of fuzzy points as follows:

$$x_{(\alpha,\beta)} = (x_\alpha, 1 - x_{1-\beta}). \tag{2.7}$$

Definition 2.7 [3]**.** Let $(X,\mathcal{T})$ and $(Y,\mathcal{U})$ be intuitionistic fuzzy topological spaces. Then a map $f : X \to Y$ is said to be
(1) *continuous* if $f^{-1}(B)$ is an intuitionistic fuzzy open set in X, for each intuitionistic fuzzy open set B in Y, or equivalently, $f^{-1}(B)$ is an intuitionistic fuzzy closed set in X, for each intuitionistic fuzzy closed set B in Y,
(2) *open* if $f(A)$ is an intuitionistic fuzzy open set in Y, for each intuitionistic fuzzy open set A in X,
(3) *closed* if $f(A)$ is an intuitionistic fuzzy closed set in Y for each intuitionistic fuzzy closed set A in X,
(4) a *homeomorphism* if f is bijective, continuous, and open.

Definition 2.8 [5]**.** A relation $\delta \subseteq I^X \times I^X$ is called a *fuzzy proximity* on X if it satisfies the following properties:
(1) $\mu\delta\rho$ implies $\rho\delta\mu$;
(2) $(\mu \vee \rho)\delta\lambda$ if and only if $\mu\delta\lambda$ or $\rho\delta\lambda$;
(3) $\mu\delta\rho$ implies $\mu \ne \tilde{0}$ and $\rho \ne \tilde{0}$;
(4) $\mu\not\delta\rho$ implies that there exists a $\lambda \subseteq I^X$ such that $\mu\not\delta\lambda$ and $1-\lambda\not\delta\rho$;
(5) $\mu \wedge \rho \ne \tilde{0}$ implies $\mu\delta\rho$.

3. Intuitionistic fuzzy proximity spaces. We are going to introduce the concept of intuitionistic fuzzy proximity spaces and continuous maps between them.

Definition 3.1. An *intuitionistic fuzzy proximity* on X is a relation δ on $I(X)$ satisfying the following properties:
(1) $A\delta B$ implies $B\delta A$;
(2) $(A \cup B)\delta C$ if and only if $A\delta C$ or $B\delta C$;
(3) $A\delta B$ implies $A \ne 0_\sim$ and $B \ne 0_\sim$;
(4) $A\not\delta B$ implies that there exists an $E \in I(X)$ such that $A\not\delta E$ and $E^c\not\delta B$;
(5) $A \cap B \ne 0_\sim$ implies $A\delta B$.
The pair (X,δ) is called an *intuitionistic fuzzy proximity space*.

We have easily the following lemma.

Lemma 3.2. *Let (X,δ) be an intuitionistic fuzzy proximity space. Then the following properties hold:*

(1) *if $A\delta B$, $A_1 \supseteq A$ and $B_1 \supseteq B$, then $A_1\delta B_1$;*

(2) *$A\delta A$ for each $A \neq 0_\sim$;*

(3) *$A\delta 1_\sim$ if and only if $A \neq 0_\sim$.*

Definition 3.3. Let (X,δ_1) and (Y,δ_2) be two intuitionistic fuzzy proximity spaces and $f : X \to Y$ a map. Then f is called a *continuous* map if $A\delta_1 B$ implies $f(A)\delta_2 f(B)$.

From the fact that $A \subseteq f^{-1}f(A)$ and $C \supseteq ff^{-1}(C)$, we obtain the following lemma.

Lemma 3.4. *Let (X,δ_1) and (Y,δ_2) be two intuitionistic fuzzy proximity spaces and $f : X \to Y$ a map. Then f is continuous if and only if $C\not\delta_2 D$ implies $f^{-1}(C)\not\delta_1 f^{-1}(D)$, for each $C,D \in I(Y)$.*

Theorem 3.5. *Let $\mathrm{cl} : I(X) \to I(X)$ be a map satisfying the conditions (1)–(4) of Theorem 2.5. Then there is a unique intuitionistic fuzzy topology $\mathcal{T}$ on X such that $\mathrm{cl} = \mathrm{cl}_{\mathcal{T}}$.*

Proof. Let $\mathcal{T} = \{A \in I(X) \mid \mathrm{cl}(A^c) = A^c\}$. First, we will show that $\mathcal{T}$ is an intuitionistic fuzzy topology on X.

(i) Clearly, $0_\sim \in \mathcal{T}$ and $1_\sim \in \mathcal{T}$.

(ii) Let $A,B \in \mathcal{T}$. Then $\mathrm{cl}(A^c) = A^c$ and $\mathrm{cl}(B^c) = B^c$. So

$$\mathrm{cl}\left((A\cap B)^c\right) = \mathrm{cl}\left(A^c \cup B^c\right) = \mathrm{cl}\left(A^c\right) \cup \mathrm{cl}\left(B^c\right) = A^c \cup B^c = (A\cap B)^c \tag{3.1}$$

and hence $A \cap B \in \mathcal{T}$.

(iii) Let $A_\alpha \in \mathcal{T}$ for all $\alpha \in \Gamma$. Then for each $\alpha \in \Gamma$, $\mathrm{cl}(A_\alpha^c) = A_\alpha^c$. Note that if $A \subseteq B$ then $\mathrm{cl}(A) \subseteq \mathrm{cl}(B)$. So

$$\mathrm{cl}\left(\left(\bigcup_\alpha A_\alpha\right)^c\right) = \mathrm{cl}\left(\bigcap_\alpha A_\alpha^c\right) \subseteq \bigcap_\alpha \mathrm{cl}\left(A_\alpha^c\right) = \bigcap_\alpha A_\alpha^c = \left(\bigcup_\alpha A_\alpha\right)^c. \tag{3.2}$$

Thus $\mathrm{cl}((\bigcup_\alpha A_\alpha)^c) = (\bigcup_\alpha A_\alpha)^c$ and hence $\bigcup_\alpha A_\alpha \in \mathcal{T}$. Therefore, $\mathcal{T}$ is an intuitionistic fuzzy topology on X.

Next, we will show that $\mathrm{cl} = \mathrm{cl}_{\mathcal{T}}$. Let $A \in I(X)$. Since $\mathrm{cl}(\mathrm{cl}(A)) = \mathrm{cl}(A)$ and $A \subseteq \mathrm{cl}(A)$,

$$\begin{aligned}\mathrm{cl}_{\mathcal{T}}(A) &= \bigcap\{F \in I(X) \mid A \subseteq F,\ F^c \in \mathcal{T}\}\\ &= \bigcap\{F \in I(X) \mid A \subseteq F,\ \mathrm{cl}(F) = F\}\\ &\subseteq \mathrm{cl}(A).\end{aligned} \tag{3.3}$$

Consider a set F such that $\mathrm{cl}(F) = F$ and $A \subseteq F$. Then $\mathrm{cl}(A) \subseteq \mathrm{cl}(F) = F$. Thus $\mathrm{cl}(A) \subseteq F$ and hence

$$\mathrm{cl}(A) \subseteq \bigcap\{F \in I(X) \mid A \subseteq F,\ \mathrm{cl}(F) = F\} = \mathrm{cl}_{\mathcal{T}}(A). \tag{3.4}$$

Hence $\mathrm{cl} = \mathrm{cl}_{\mathcal{T}}$.

Finally, we will show that such a $\mathcal{T}$ is unique. Suppose $\mathcal{T}^*$ is an intuitionistic fuzzy topology on X such that $\mathrm{cl} = \mathrm{cl}_{\mathcal{T}^*}$. Let $A \in \mathcal{T}$. Then $\mathrm{cl}(A^c) = A^c$. So $\mathrm{cl}_{\mathcal{T}^*}(A^c) = \mathrm{cl}(A^c) = A^c$ and hence $A \in \mathcal{T}^*$. Also, let $A \in \mathcal{T}^*$. Then $\mathrm{cl}(A^c) = \mathrm{cl}_{\mathcal{T}^*}(A^c) = A^c$ and hence $A \in \mathcal{T}$. Thus $\mathcal{T} = \mathcal{T}^*$. □

Theorem 3.6. *Let (X,δ) be an intuitionistic fuzzy proximity space and define a map $\mathrm{cl}: I(X) \to I(X)$ by*

$$\mathrm{cl}(A) = \bigcap \{B^c \in I(X) \mid A \not\delta B\}, \tag{3.5}$$

for each $A \in I(X)$. Then the following properties hold:
(1) $A \subseteq \mathrm{cl}(A)$;
(2) $\mathrm{cl}(\mathrm{cl}(A)) = \mathrm{cl}(A)$;
(3) $\mathrm{cl}(A \cup B) = \mathrm{cl}(A) \cup \mathrm{cl}(B)$;
(4) $\mathrm{cl}(0_\sim) = 0_\sim$.

Proof. (1) Let $A = (\mu_A, \gamma_A) \in I(X)$. Take any $B = (\mu_B, \gamma_B) \in I(X)$ such that $A \not\delta B$. Then $A \cap B = 0_\sim = (\tilde{0}, \tilde{1})$ and hence $\mu_A \wedge \mu_B = 0$ and $\gamma_A \vee \gamma_B = 1$. So $\mu_A + \mu_B \leq 1$ and $\gamma_A + \gamma_B \geq 1$. Thus $\mu_B \leq 1 - \gamma_B \leq \gamma_A$ and $\gamma_B \geq 1 - \gamma_A \geq \mu_A$. Hence $B^c = (\gamma_B, \mu_B) \supseteq (\mu_A, \gamma_A) = A$. Therefore,

$$A \subseteq \bigcap \{B^c \mid A \not\delta B\} = \mathrm{cl}(A). \tag{3.6}$$

(2) It is sufficient to show that $\mathrm{cl}(A) \not\delta B$ if and only if $A \not\delta B$. If $A \delta B$, then $\mathrm{cl}(A) \delta B$ obviously. Conversely, suppose that $A \not\delta B$ and $\mathrm{cl}(A) \delta B$. Then there exists an $E \in I(X)$ such that $B \not\delta E$ and $E^c \not\delta A$. Since $\mathrm{cl}(A) \delta B$ and $E \not\delta B$, $\mathrm{cl}(A) \nsubseteq E$ and hence $\mu_{\mathrm{cl}(A)} \nleq \mu_E$ or $\gamma_{\mathrm{cl}(A)} \ngeq \gamma_E$. So there exists an $x \in X$ such that

$$\mu_{\mathrm{cl}(A)}(x) > \mu_E(x) \text{ or } \gamma_{\mathrm{cl}(A)}(x) < \gamma_E(x). \tag{3.7}$$

If $\mu_{\mathrm{cl}(A)}(x) > \mu_E(x)$, we choose $a \in I$ such that $\mu_E(x) < a < \mu_{\mathrm{cl}(A)}(x)$. Define $G: X \to I \times I$ by

$$G(y) = \begin{cases} (0,a) & \text{if } y = x, \\ (0,1) & \text{if } y \neq x. \end{cases} \tag{3.8}$$

Then $G \in I(X)$ and $G \subseteq E^c$. If $G \not\delta A$ then $\mathrm{cl}(A) \subseteq G^c$ by the definition of closure and hence $\mu_{\mathrm{cl}(A)}(x) \leq \mu_{G^c}(x) = \gamma_G(x) = a < \mu_{\mathrm{cl}(A)}(x)$. This is a contradiction. Thus $G \delta A$. Since $G \subseteq E^c$, $A \delta E^c$. This is a contradiction to the fact that $E^c \not\delta A$. Hence $\mu_{\mathrm{cl}(A)}(x) \leq \mu_E(x)$.

Next, if $\gamma_{\mathrm{cl}(A)}(x) < \gamma_E(x)$, we can choose $b \in I$ such that $\gamma_{\mathrm{cl}(A)}(x) < b < \gamma_E(x)$. Define $H: X \to I \times I$ by

$$H(y) = \begin{cases} (b, 1-b) & \text{if } y = x, \\ (0,1) & \text{if } y \neq x. \end{cases} \tag{3.9}$$

Then $H \in I(X)$. Since $\mu_H(x) = b < \gamma_E(x) = \mu_{E^c}(x)$ and $\gamma_H(x) = 1 - b > 1 - \gamma_E(x) \geq \mu_E(x) = \gamma_{E^c}(x)$, $H \subseteq E^c$. If $H \not\delta A$ then $\mathrm{cl}(A) \subseteq H^c$ by the definition of closure and hence $\gamma_{\mathrm{cl}(A)}(x) \geq \gamma_{H^c}(x) = \mu_H(x) = b > \gamma_{\mathrm{cl}(A)}(x)$. This is a contradiction. Thus we have $H\delta A$. Since $H \subseteq E^c$, $A\delta E^c$. This is a contradiction. In any case, we have a contradiction. So $A \not\delta B$ implies $\mathrm{cl}(A) \not\delta B$.

(3) It is easy to show that $\mathrm{cl}(A \cup B) \supseteq \mathrm{cl}(A) \cup \mathrm{cl}(B)$. On the other hand, suppose $(\mu_{\mathrm{cl}(A\cup B)}, \gamma_{\mathrm{cl}(A\cup B)}) = \mathrm{cl}(A \cup B) \nsubseteq \mathrm{cl}(A) \cup \mathrm{cl}(B) = (\mu_{\mathrm{cl}(A)} \vee \mu_{\mathrm{cl}(B)}, \gamma_{\mathrm{cl}(A)} \wedge \gamma_{\mathrm{cl}(B)})$. Then there exists an $x \in X$ such that

$$\mu_{\mathrm{cl}(A\cup B)}(x) > \mu_{\mathrm{cl}(A)}(x) \vee \mu_{\mathrm{cl}(B)}(x) \text{ or } \gamma_{\mathrm{cl}(A\cup B)}(x) < \gamma_{\mathrm{cl}(A)}(x) \wedge \gamma_{\mathrm{cl}(B)}(x). \tag{3.10}$$

Case 1. Suppose $\mu_{\mathrm{cl}(A\cup B)}(x) > \mu_{\mathrm{cl}(A)}(x) \vee \mu_{\mathrm{cl}(B)}(x)$. We may assume $\mu_{\mathrm{cl}(A)}(x) \geq \mu_{\mathrm{cl}(B)}(x)$. Let $\mu_{\mathrm{cl}(A\cup B)}(x) = a$. Then $\mu_{\mathrm{cl}(A)}(x) < a$ and hence there exists an $\epsilon > 0$ such that $\mu_{\mathrm{cl}(A)}(x) < a - \epsilon$. Since $\mu_{\mathrm{cl}(A)}(x) = \bigwedge\{\gamma_C(x) \mid C \not\delta A\}$, there exists a $C \in I(X)$ such that $C \not\delta A$ and $\gamma_C(x) < a - \epsilon$. Note that

$$\gamma_C(x) \geq \mu_{\mathrm{cl}(A)}(x) \geq \mu_{\mathrm{cl}(B)}(x) > \mu_{\mathrm{cl}(B)}(x) - \frac{\epsilon}{2}, \tag{3.11}$$

and hence $\mu_{\mathrm{cl}(B)}(x) < \gamma_C(x) + \epsilon/2$. Since $\mu_{\mathrm{cl}(B)}(x) = \bigwedge\{\gamma_D(x) \mid D \not\delta B\}$, there exists a $D \in I(X)$ such that $D \not\delta B$ and $\gamma_D(x) < \gamma_C(x) + \epsilon/2$. Since $(C \cap D) \not\delta A$ and $(C \cap D) \not\delta B$, we have $(C \cap D) \not\delta (A \cup B)$. So, from the definition of closure, we have $\mathrm{cl}(A \cup B) \subseteq (C \cap D)^c$. Also $\gamma_C(x) \vee \gamma_D(x) < \gamma_C(x) + \epsilon/2$. Hence

$$\begin{aligned} a = \mu_{\mathrm{cl}(A\cup B)}(x) &\leq \mu_{(C\cap D)^c}(x) = \gamma_{C\cap D}(x) = \gamma_C(x) \vee \gamma_D(x) \\ &< \gamma_C(x) + \frac{\epsilon}{2} < a - \epsilon + \frac{\epsilon}{2} = a - \frac{\epsilon}{2}. \end{aligned} \tag{3.12}$$

This is a contradiction.

Case 2. Suppose $\gamma_{\mathrm{cl}(A\cup B)}(x) < \gamma_{\mathrm{cl}(A)}(x) \wedge \gamma_{\mathrm{cl}(B)}(x)$. We may assume $\gamma_{\mathrm{cl}(A)}(x) \leq \gamma_{\mathrm{cl}(B)}(x)$. Let $\gamma_{\mathrm{cl}(A\cup B)}(x) = a$. Then $a < \gamma_{\mathrm{cl}(A)}(x)$ and hence there exists an $\epsilon > 0$ such that $a + \epsilon < \gamma_{\mathrm{cl}(A)}(x)$. Since $\gamma_{\mathrm{cl}(A)}(x) = \bigvee\{\mu_C(x) \mid C \not\delta A\}$, there exists an intuitionistic fuzzy set $C \in I(X)$ such that $C \not\delta A$ and $a + \epsilon < \mu_C(x)$. Note that,

$$\mu_C(x) \leq \gamma_{\mathrm{cl}(A)}(x) \leq \gamma_{\mathrm{cl}(B)}(x) < \gamma_{\mathrm{cl}(B)}(x) + \frac{\epsilon}{2}, \tag{3.13}$$

and hence $\mu_C(x) - \epsilon/2 < \gamma_{\mathrm{cl}(B)}(x)$. Since $\gamma_{\mathrm{cl}(B)}(x) = \bigvee\{\mu_D(x) \mid D \not\delta B\}$, there exists a $D \in I(X)$ such that $D \not\delta B$ and $\mu_C(x) - \epsilon/2 < \mu_D(x)$. Since $(C \cap D) \not\delta A$ and $(C \cap D) \not\delta B$, we have $(C \cap D) \not\delta (A \cup B)$. So, from the definition of closure, we have $\mathrm{cl}(A \cup B) \subseteq (C \cap D)^c$. Also $\mu_C(x) - \epsilon/2 < \mu_C(x) \wedge \mu_D(x)$. Hence

$$\begin{aligned} a = \gamma_{\mathrm{cl}(A\cup B)}(x) &\geq \gamma_{(C\cap D)^c}(x) = \mu_{C\cap D}(x) = \mu_C(x) \wedge \mu_D(x) \\ &> \mu_C(x) - \frac{\epsilon}{2} > a + \epsilon - \frac{\epsilon}{2} = a + \frac{\epsilon}{2}. \end{aligned} \tag{3.14}$$

This is a contradiction.

In any case, we have a contradiction. Therefore $\mathrm{cl}(A \cup B) = \mathrm{cl}(A) \cup \mathrm{cl}(B)$.

(4) Since $1_\sim \not\delta 0_\sim$, $\mathrm{cl}(0_\sim) = \bigcap\{B^c \mid B \not\delta 0_\sim\} = (0,1) = 0_\sim$. □

Theorem 3.7. *For an intuitionistic fuzzy proximity space* (X,δ)*, the family*

$$\mathcal{T}(\delta) = \{A \in I(X) \mid \mathrm{cl}(A^c) = A^c\} \tag{3.15}$$

is an intuitionistic fuzzy topology on X*.*

Proof. By Theorems 3.5 and 3.6, the proof follows. □

Definition 3.8. The topology $\mathcal{T}(\delta)$ defined in Theorem 3.7 is called the intuitionistic fuzzy topology on X induced by the fuzzy proximity δ.

Theorem 3.9. *Let* (X,δ_1) *and* (Y,δ_2) *be two intuitionistic fuzzy proximity spaces and* $f : X \to Y$ *a continuous map. Then* $f : (X,\mathcal{T}(\delta_1)) \to (Y,\mathcal{T}(\delta_2))$ *is continuous with respect to the corresponding intuitionistic fuzzy topologies* $\mathcal{T}(\delta_1)$ *and* $\mathcal{T}(\delta_2)$*.*

Proof. Let $A \in \mathcal{T}(\delta_2)$. Then $\mathrm{cl}(A^c) = A^c$. We will show that $\mathrm{cl}(f^{-1}(A)^c) = f^{-1}(A)^c$. Clearly $f^{-1}(A)^c \subseteq \mathrm{cl}(f^{-1}(A)^c)$. Conversely, let $B\not\delta_2 A^c$. Since f is continuous, $f^{-1}(B)\not\delta_1 f^{-1}(A^c) = f^{-1}(A)^c$. Thus

$$\mathrm{cl}(f^{-1}(A)^c) = \bigcap \{K^c \mid K\not\delta_1 f^{-1}(A)^c\} \subseteq f^{-1}(B)^c. \tag{3.16}$$

Hence for any $B\not\delta_2 A^c$, $\mathrm{cl}(f^{-1}(A)^c) \subseteq f^{-1}(B)^c$. So, we have

$$\begin{aligned}
\mathrm{cl}(f^{-1}(A)^c) &\subseteq \bigcap \{f^{-1}(B)^c \mid B\not\delta_2 A^c\} \\
&= \bigcap \{f^{-1}(B^c) \mid B\not\delta_2 A^c\} \\
&= f^{-1}\Big(\bigcap \{B^c \mid B\not\delta_2 A^c\}\Big) \\
&= f^{-1}(\mathrm{cl}(A^c)) = f^{-1}(A^c) = f^{-1}(A)^c.
\end{aligned} \tag{3.17}$$

Thus $\mathrm{cl}(f^{-1}(A)^c) = f^{-1}(A)^c$. Hence $f^{-1}(A)$ is open. Therefore, f is a continuous map. □

4. The δ-neighborhood in the intuitionistic fuzzy proximity. In this section, we will introduce the notion of the δ-neighborhood in the intuitionistic fuzzy proximity.

Definition 4.1. Let (X,δ) be an intuitionistic fuzzy proximity space. For $A, B \in I(X)$, the intuitionistic fuzzy set B is said to be a *δ-neighborhood* of A (in symbols $A \ll B$) if $A\not\delta B^c$.

Clearly, we know that if $A \ll B$, then $A \subseteq B$.

Theorem 4.2. *Let* (X,δ) *be an intuitionistic fuzzy proximity space and* $A, B \in I(X)$*. Then the following properties hold:*

(1) $A \ll B$ *if and only if* $\mathrm{cl}(A) \ll B$;

(2) *if* $A \ll B$*, then there exists an element* G *of the intuitionistic fuzzy topology* $\mathcal{T}(\delta)$ *induced by* δ *on* X *such that* $A \subseteq G \subseteq B$;

(3) *if* $A\not\delta B$*, then there are* $E, F \in I(X)$ *such that* $A \ll E$*,* $B \ll F$*, and* $E\not\delta F$*.*

Proof. (1) It follows from the fact that $A\bar{\delta}B$ if and only if $\mathrm{cl}(A)\bar{\delta}B$ (see the proof of Theorem 3.6(2)).

(2) Let $A \ll B$. Then $A\bar{\delta}B^c$ and hence

$$\mathrm{cl}\left(B^c\right) = \bigcap \{K^c \mid K\bar{\delta}B^c\} \subseteq A^c. \tag{4.1}$$

Thus $B^c \subseteq \mathrm{cl}(B^c) \subseteq A^c$. Put $G = \mathrm{cl}(B^c)^c$. Note that

$$\mathrm{cl}\left(G^c\right) = \mathrm{cl}\left(\mathrm{cl}\left(B^c\right)\right) = \mathrm{cl}\left(B^c\right) = G^c. \tag{4.2}$$

Hence $G \in \mathcal{T}(\delta)$ and $A \subseteq G \subseteq B$.

(3) Suppose $A\bar{\delta}B$. Then there exists an $E \in I(X)$ such that $A\bar{\delta}E^c$ and $E\bar{\delta}B$. Since $B\bar{\delta}E$, there exists an $F \in I(X)$ such that $B\bar{\delta}F^c$ and $F\bar{\delta}E$. Thus there are $E,F \in I(X)$ such that $A \ll E$, $B \ll F$, and $E\bar{\delta}F$. □

Theorem 4.3. *Let (X,δ) be an intuitionistic fuzzy proximity space. Then the relation $\ll$ on $I(X)$ has the following properties:*

(1) $1_\sim \ll 1_\sim$;
(2) $A \ll B$ *implies* $A \cap B^c = 0_\sim$;
(3) *if* $A_1 \subseteq A \ll B \subseteq B_1$, *then* $A_1 \ll B_1$;
(4) $A \ll B_1 \cap B_2$ *if and only if* $A \ll B_1$ *and* $A \ll B_2$;
(5) $A \ll B$ *implies* $B^c \ll A^c$;
(6) *if* $A \ll B$, *then there exists a set* $E \in I(X)$ *such that* $A \ll E \ll B$.

Proof. (1) Since $1_\sim\bar{\delta}0_\sim = 1_\sim^c$, we have $1_\sim \ll 1_\sim$.

(2) Let $A \ll B$. Then $A\bar{\delta}B^c$ and hence $A \cap B^c = 0_\sim$.

(3) Let $A_1 \subseteq A \ll B \subseteq B_1$. Then $A\bar{\delta}B^c$. Since $A_1 \subseteq A$ and $B_1^c \subseteq B^c$, we have $A_1\bar{\delta}B_1^c$ and hence $A_1 \ll B_1$.

(4)

$$\begin{aligned} A \ll B_1 \cap B_2 &\Longleftrightarrow A\bar{\delta}\left(B_1 \cap B_2\right)^c = B_1^c \cup B_2^c \\ &\Longleftrightarrow A\bar{\delta}B_1^c \text{ and } A\bar{\delta}B_2^c \\ &\Longleftrightarrow A \ll B_1 \text{ and } A \ll B_2. \end{aligned} \tag{4.3}$$

(5) Let $A \ll B$. Then $A\bar{\delta}B^c$ and hence $B^c\bar{\delta}A = (A^c)^c$. Thus $B^c \ll A^c$.

(6) Let $A \ll B$. Then $A\bar{\delta}B^c$. Thus there exists a set $E \in I(X)$ such that $A\bar{\delta}E^c$ and $E\bar{\delta}B^c$ and hence $A \ll E \ll B$. □

Theorem 4.4. *Let $\ll$ be a relation on $I(X)$ satisfying (1)–(6) of the above theorem. Then the relation δ on $I(X)$, defined by $A\bar{\delta}B$ if and only if $A \ll B^c$, is an intuitionistic fuzzy proximity on X. Also, with respect to this intuitionistic fuzzy proximity, B is a δ-neighborhood of A if and only if $A \ll B$.*

Proof. First, we will show that δ is an intuitionistic fuzzy proximity on X.

(1) Let $A\bar{\delta}B$. Then $A \ll B^c$ and hence $B \ll A^c$. So $B\bar{\delta}A$.

(2)

$$\begin{aligned} A\bar{\delta}(B_1 \cup B_2) &\Longleftrightarrow A \ll (B_1 \cup B_2)^c = B_1^c \cap B_2^c \\ &\Longleftrightarrow A \ll B_1^c \text{ and } A \ll B_2^c \\ &\Longleftrightarrow A\bar{\delta}B_1 \text{ and } A\bar{\delta}B_2. \end{aligned} \tag{4.4}$$

(3) Let $B = 0_\sim$. Since $A \subseteq 1_\sim \ll 1_\sim \subseteq 1_\sim$, we have $A \ll 1_\sim = (0_\sim)^c = B^c$ and hence $A\bar{\delta}B$.

(4) Let $A\bar{\delta}B$. Then $A \ll B^c$. Thus there exists an $E \in I(X)$ such that $A \ll E \ll B^c$. So $A \ll (E^c)^c$ and $E \ll B^c$. Hence $A\bar{\delta}E^c$ and $E\bar{\delta}B$.

(5) If $A\bar{\delta}B$, then $A \ll B^c$ and hence $A \cap B = A \cap (B^c)^c = 0_\sim$.

Therefore δ is an intuitionistic fuzzy proximity on X. Clearly, B is a δ-neighborhood of A if and only if $A\bar{\delta}B^c$ if and only if $A \ll (B^c)^c = B$. □

Theorem 4.5. *Let (X, δ) be an intuitionistic fuzzy proximity space and $A \in I(X)$. Then*

$$\text{cl}(A) = \bigcap \{B \mid A \ll B\}. \tag{4.5}$$

Proof. Let $K = \bigcap\{B \mid A \ll B\}$. Take any B with $A \ll B$. Then $\text{cl}(A) \ll B$ and hence $\text{cl}(A) \subseteq B$. Thus

$$\text{cl}(A) \subseteq \bigcap \{B \mid A \ll B\} = K. \tag{4.6}$$

Conversely, suppose $\text{cl}(A) \nsupseteq K$. Then $\mu_{\text{cl}(A)} \ngeq \mu_K$ or $\gamma_{\text{cl}(A)} \nleq \gamma_K$ and hence there exists an $x \in X$ such that

$$\mu_{\text{cl}(A)}(x) < \mu_K(x) \text{ or } \gamma_{\text{cl}(A)}(x) > \gamma_K(x). \tag{4.7}$$

Suppose $\mu_{\text{cl}(A)}(x) < \mu_K(x)$. Let $\mu_K(x) = a$. Then there exists an $\epsilon > 0$ such that $\mu_{\text{cl}(A)}(x) < a - \epsilon$. Since $\mu_{\text{cl}(A)}(x) = \bigwedge\{\gamma_B(x) \mid A\bar{\delta}B\}$, there exists an $E \in I(X)$ such that $A\bar{\delta}E$ and $\gamma_E(x) < a - \epsilon$. Since $A\bar{\delta}(E^c)^c$, $A \ll E^c$ and hence $K = \bigcap\{B \mid A \ll B\} \subseteq E^c$. Thus

$$a = \mu_K(x) \leq \mu_{E^c}(x) = \gamma_E(x) < a - \epsilon. \tag{4.8}$$

This is a contradiction. Next, suppose $\gamma_{\text{cl}(A)}(x) > \gamma_K(x)$. Let $\gamma_K(x) = a$. Then there exists an $\epsilon > 0$ such that $\gamma_{\text{cl}(A)}(x) > a + \epsilon$. Since $\gamma_{\text{cl}(A)}(x) = \bigvee\{\mu_B(x) \mid A\bar{\delta}B\}$, there exists an $F \in I(X)$ such that $A\bar{\delta}F$ and $\mu_F(x) > a + \epsilon$. Since $A\bar{\delta}(F^c)^c$, $A \ll F^c$ and hence $K = \bigcap\{B \mid A \ll B\} \subseteq F^c$. Thus

$$a = \gamma_K(x) \geq \gamma_{F^c}(x) = \mu_F(x) > a + \epsilon. \tag{4.9}$$

This is a contradiction. □

5. Category of intuitionistic fuzzy proximity spaces. We knew the relationship between fuzzy topological spaces and fuzzy proximity spaces (see [5, 6]). The relationship between fuzzy topological spaces and intuitionistic fuzzy topological spaces had been studied in [8]. Also we have had the relationship between intuitionistic fuzzy proximity spaces and intuitionistic fuzzy topological spaces in Theorems 3.7 and 3.9. Now, we are going to find a categorical relationship between fuzzy proximity spaces and intuitionistic fuzzy proximity spaces.

Let **FProx** be the category of all fuzzy proximity spaces and proximity maps and **IFProx** the category of all intuitionistic fuzzy proximity spaces and continuous maps.

Theorem 5.1. *Define* $F:\mathbf{FProx}\to\mathbf{IFProx}$ *by*

$$F(X,\sigma)=(X,\delta),\qquad F(f)=f, \tag{5.1}$$

where for $A,B\in I(X)$, $A\delta B$ *if and only if* $(1-\gamma_A)\sigma(1-\gamma_B)$. *Then* F *is a functor.*

Proof. First, we will show that δ is an intuitionistic fuzzy proximity on X.

(1) Clearly, $A\delta B$ implies $B\delta A$.

(2) Note that $1-\gamma_{A\cup B}=1-(\gamma_A\wedge\gamma_B)=(1-\gamma_A)\vee(1-\gamma_B)$. Thus

$$\begin{aligned} C\delta(A\cup B) &\Longleftrightarrow (1-\gamma_C)\sigma(1-\gamma_{A\cup B})\\ &\Longleftrightarrow (1-\gamma_C)\sigma[(1-\gamma_A)\vee(1-\gamma_B)]\\ &\Longleftrightarrow (1-\gamma_C)\sigma(1-\gamma_A)\text{ or }(1-\gamma_C)\sigma(1-\gamma_B)\\ &\Longleftrightarrow C\delta A\text{ or }C\delta B. \end{aligned} \tag{5.2}$$

(3) Since $(1-\gamma_A)\not\sigma\tilde{0}=(1-\gamma_{0_\sim})$, we have $A\not\delta 0_\sim$.

(4) Let $A\not\delta B$. Then $(1-\gamma_A)\not\sigma(1-\gamma_B)$. Then there exists a $\rho\in I^X$ such that $(1-\gamma_A)\not\sigma\rho$ and $(1-\rho)\not\delta(1-\gamma_B)$. Let $E=(\mu_E,\gamma_E)=(\rho,1-\rho)$. Then $E\in I(X)$. Since $(1-\gamma_A)\not\sigma\rho=(1-\gamma_E)$, $A\not\delta E$. Note that $1-\gamma_{E^c}=1-\mu_E=1-\rho$ and $(1-\rho)\not\sigma(1-\gamma_B)$. So $E^c\not\delta B$.

(5) Let $A\cap B\neq 0_\sim$. Suppose $\gamma_A\vee\gamma_B=\tilde{1}$. Then

$$\tilde{1}=\gamma_A\vee\gamma_B\le(1-\mu_A)\vee(1-\mu_B)=1-(\mu_A\wedge\mu_B) \tag{5.3}$$

and hence $\mu_A\wedge\mu_B=\tilde{0}$. So $A\cap B=0_\sim$. This is a contradiction. Thus $\gamma_A\vee\gamma_B\neq 1$. Hence $(1-\gamma_A)\wedge(1-\gamma_B)=1-(\gamma_A\vee\gamma_B)\neq 0$. Thus $(1-\gamma_A)\sigma(1-\gamma_B)$ and hence $A\delta B$. Therefore (X,δ) is an intuitionistic fuzzy proximity space.

Next, we will show that if $f:(X,\sigma_1)\to(Y,\sigma_2)$ is a proximity map, then $f:(X,\delta_1)\to(Y,\delta_2)$ is a continuous map. Let $C,D\in I(Y)$ and $C\not\delta_2 D$. Then $(1-\gamma_C)\not\sigma_2(1-\gamma_D)$. Since $f:(X,\sigma_1)\to(Y,\sigma_2)$ is a proximity map, $f^{-1}(1-\gamma_C)\not\sigma_1 f^{-1}(1-\gamma_D)$. Note that

$$\begin{aligned} f^{-1}(1-\gamma_C)&=1-f^{-1}(\gamma_C)=1-\gamma_{f^{-1}(C)},\\ f^{-1}(1-\gamma_D)&=1-f^{-1}(\gamma_D)=1-\gamma_{f^{-1}(D)}. \end{aligned} \tag{5.4}$$

So $[1-\gamma_{f^{-1}(C)}]\not\sigma_1[1-\gamma_{f^{-1}(D)}]$ and hence $f^{-1}(C)\not\delta_1 f^{-1}(D)$. Therefore $f:(X,\delta_1)\to(Y,\delta_2)$ is a continuous map. In all, F is a functor. □

Theorem 5.2. *Define* $G : \mathbf{IFProx} \to \mathbf{FProx}$ *by*

$$G(X,\delta) = (X,\sigma), \qquad G(f) = f, \tag{5.5}$$

where for $\nu,\rho \in I^X$, $\nu\sigma\rho$ *if and only if* $(0,1-\nu)\delta(0,1-\rho)$. *Then* G *is a functor.*

Proof. First, we will show that σ is a fuzzy proximity on X.

(1) Clearly, if $\nu\sigma\rho$, then $\rho\sigma\nu$.

(2) Note that $(0,1-(\nu\vee\rho)) = (0,(1-\nu)\wedge(1-\rho)) = (0,1-\nu)\cup(0,1-\rho)$. Thus

$$\begin{aligned} \lambda\sigma(\nu\vee\rho) &\Longleftrightarrow (0,1-\lambda)\delta(0,1-(\nu\vee\rho)) \\ &\Longleftrightarrow (0,1-\lambda)\delta[(0,1-\nu)\cup(0,1-\rho)] \\ &\Longleftrightarrow (0,1-\lambda)\delta(0,1-\nu) \text{ or } (0,1-\lambda)\delta(0,1-\rho) \\ &\Longleftrightarrow \lambda\sigma\nu \text{ or } \lambda\sigma\rho. \end{aligned} \tag{5.6}$$

(3) Since $(0,1-\nu)\not\delta 0_\sim = (0,1-0)$, we have $\nu\not\sigma\tilde{0}$.

(4) Let $\nu\not\sigma\rho$. Then $(0,1-\nu)\not\delta(0,1-\rho)$. So there exists a set $E = (\mu_E,\gamma_E) \in I(X)$ such that

$$(0,1-\nu)\not\delta(\mu_E,\gamma_E), \qquad (\gamma_E,\mu_E)\not\delta(0,1-\rho). \tag{5.7}$$

Since $(0,1-\mu_E) \subseteq (\mu_E,\gamma_E)$ and $(0,\mu_E) \subseteq (\gamma_E,\mu_E)$,

$$(0,1-\nu)\not\delta(0,1-\mu_E), \qquad (0,\mu_E)\not\delta(0,1-\rho). \tag{5.8}$$

Thus there exists a set $\mu_E \in I^X$ such that $\nu\not\sigma\mu_E$ and $1-\mu_E\not\sigma\rho$.

(5) Let $\nu\wedge\rho \neq \tilde{0}$. Then

$$(0,1-\nu)\cap(0,1-\rho) = (0,1-(\nu\wedge\rho)) \neq 0_\sim. \tag{5.9}$$

So $(0,1-\nu)\delta(0,1-\rho)$ and hence $\nu\sigma\rho$. Therefore (X,σ) is a fuzzy proximity space.

Next, we will show that if $f : (X,\delta_1) \to (Y,\delta_2)$ is a continuous map then $f : (X,\sigma_1) \to (Y,\sigma_2)$ is a proximity map. Let $\nu,\rho \in I^Y$ and $\nu\not\sigma_2\rho$. Then $(0,1-\nu)\not\delta_2(0,1-\rho)$. Since $f : (X,\delta_1) \to (Y,\delta_2)$ is a continuous map, $f^{-1}((0,1-\nu))\not\delta_1 f^{-1}((0,1-\rho))$. Note that

$$\begin{aligned} f^{-1}((0,1-\nu)) &= (f^{-1}(0),f^{-1}(1-\nu)) = (0,1-f^{-1}(\nu)), \\ f^{-1}((0,1-\rho)) &= (f^{-1}(0),f^{-1}(1-\rho)) = (0,1-f^{-1}(\rho)). \end{aligned} \tag{5.10}$$

So $(0,1-f^{-1}(\nu))\not\delta_1(0,1-f^{-1}(\rho))$ and hence $f^{-1}(\nu)\not\sigma_1 f^{-1}(\rho)$. Thus $f : (X,\sigma_1) \to (Y,\sigma_2)$ is a proximity map. In all, G is a functor. □

Theorem 5.3. *The functor* $F : \mathbf{FProx} \to \mathbf{IFProx}$ *is a left adjoint of the functor* $G : \mathbf{IFProx} \to \mathbf{FProx}$.

Proof. First, we will show that for any $(X,\sigma) \in \mathbf{FProx}$, $1_X : (X,\sigma) \to G(F(X,\sigma))$ is a proximity map. Let $\nu,\rho \in I^X$ and $\nu\sigma\rho$. Then $(0,1-\nu)F(\sigma)(0,1-\rho)$ and hence $\nu G(F(\sigma))\rho$.

Next, consider $(Y,\delta) \in$ **IFProx** and a proximity map $f : (X,\sigma) \to G(Y,\delta)$. In order to show that $f : F(X,\sigma) \to (Y,\delta)$ is a continuous map, let $A,B \subseteq I(Y)$ and $A\not\delta B$. Since $(0,\gamma_A) \subseteq (\mu_A,\gamma_A) = A$ and $(0,\gamma_B) \subseteq (\mu_B,\gamma_B) = B$, $(0,\gamma_A)\not\delta(0,\gamma_B)$ and hence $(1-\gamma_A)G(\not\delta)(1-\gamma_B)$. Since $f : (X,\sigma) \to G(Y,\delta)$ is a proximity map, $f^{-1}(1-\gamma_A)\not\sigma f^{-1}(1-\gamma_B)$. Note that

$$f^{-1}(1-\gamma_A) = 1-f^{-1}(\gamma_A) = 1-\gamma_{f^{-1}(A)},$$
$$f^{-1}(1-\gamma_B) = 1-f^{-1}(\gamma_B) = 1-\gamma_{f^{-1}(B)}. \tag{5.11}$$

So $[1-\gamma_{f^{-1}(A)}]\not\sigma[1-\gamma_{f^{-1}(B)}]$ and hence $f^{-1}(A)F(\not\sigma)f^{-1}(B)$. Thus $f : F(X,\sigma) \to (Y,\delta)$ is a continuous map. Therefore 1_X is a G-universal map for (X,σ) in **FProx**. □

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Seok Jong Lee: Department of Mathematics, Chungbuk National University, Cheongju 361-763, Korea
E-mail address: sjl@cbnu.ac.kr

Eun Pyo Lee: Department of Mathematics, Seonam University, Namwon 590-711, Korea
E-mail address: eplee@seonam.ac.kr