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# ON A FIXED POINT THEOREM OF PATHAK

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ABSTRACT. It is shown that the continuity of the mapping in Pathak's fixed point theorem for normed spaces is not necessary.

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## 1. INTRODUCTION AND MAIN RESULTS.

In [1] Pathak gives the following definitions:

DEFINITION 1. Let X be a normed vector space; then T, a self mapping of X is called a 'generalized contractive mapping' if

$$\|Tx-Ty\| \leq q \max\Big\{\|x-y\|, \frac{\|x-Tx\|[1-\|x-Ty\|]}{1+\|x-Tx\|}, \frac{\|x-Ty\|[1-\|x-Tx\|]}{1+\|x-Ty\|}, \frac{\|y-Tx\|[1-\|y-Ty\|}{1+\|Tx-y\|}, \frac{\|y-Ty\|[1-\|Tx-y\|]}{1+\|y-Ty\|}, \qquad (1.1)$$

for all $x,y$ in $X$, where $0<q<1$.

DEFINITION 2. Let T be a self mapping of a Banach space X. The Mann iterative process associated with T is defined in the following manner:

Let $x_0$ be in $X$ and set $x_{n+1}=(1-c_n)x_n+c_nTx_n$, for $n\geq 0$, where $c_n$ satisfies (i) $c_o=1$, (ii) $0<c_n<1$ for $n>0$, (iii) $\lim_{n\to\infty}c_n=h>0$.

He then proves the following theorem:

THEOREM. Let X be a closed, convex subset of a normed space N, let T be a generalized contractive mapping of X with T continuous on X, and let $\{x_n\}$, the sequence of Mann iterates associated with T, be the same as defined above where $\{c_n\}$ satisfies (i), (ii) and (iii). If $\{x_n\}$ converges in X, then it converges to a fixed point of T.

Pathak finally asks if the continuity of T is necessary in the theorem for T to have a fixed point.

The answer is in the affirmative. To see this, note that if T is a generalized contractive mapping then T also satisfies the inequality

$$\|Tx - Ty\| \le q \max\{\|x-y\|, \|x-Tx\|, \|x-Ty\|, \|y-Tx\|, \|y-Ty\|\} \quad (1.2)$$

for all $x, y$ in $X$, where $0 < q < 1$.

Using inequality (1.2) now instead of inequality (1.1) to simplify the work, it follows in exactly the same way as in Pathak's proof of the theorem that if $\lim_{n\to\infty} x_n = z$, then

$$\|z - Tz\| \le \|z - x_{n+1}\| + (1-c_n)\|x_n - Tz\|$$
$$+ c_n q \max\{\|x_n - z\|, \|x_n - Tx_n\|, \|x_n - Tz\|, \|Tx_n - z\|, \|z - Tz\|\} \quad (1.3)$$

It now follows from the definition of $x_n$ in Definition 2 that

$$Tx_n = \frac{x_{n+1} - (1-c_n)x_n}{c_n}$$

and so

$$\lim_{n\to\infty} Tx_n = z.$$

On letting $n$ tend to infinity in inequality (1.3) we now have

$$\|z - Tz\| \le (1-h)\|z - Tz\| + hq \max\{0, \|z - Tz\|\}$$

$$= (1 - h + hq)\|z - Tz\|,$$

where $1 - h + hq < 1$. Thus $Tz = z$.

## REFERENCES

1. PATHAK, H.K., Some Fixed Point Theorems on Contractive Mappings, Bull. Cal. Math. Soc. 80, 183.