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A COUPLED MAGNETO-THERMO-ELASTIC PROBLEM IN A PERFECTLY CONDUCTING ELASTIC HALF-SPACE WITH THERMAL RELAXATION

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ABSTRACT. In the present paper we consider the magneto-thermo-elastic wave produced by a thermal shock in a perfectly conducting elastic half-space. Here the Lord-Shulman theory of thermoelasticity [1] is used to account for the interaction between the elastic and thermal fields. The solution obtained in analytical form reduces to those of Kaliski and Nowacki [2] when the coupling between the temperature and strain fields and the relaxation time are neglected. The results also agree with those of Massalas and DaLamangas [3] in absence of the thermal relaxation time.

KEY WORDS AND PHRASES. Magneto-thermoelastic wave; Thermal relaxation time.
1980 AMS SUBJECT CLASSIFICATION CODE.

1. INTRODUCTION.

Kaliski and Nowacki [2] investigated the problem of magneto-thermo-elastic disturbances generated by a thermal shock in a perfectly conducting elastic half-space in contact with a vacuum. It was assumed that both in the medium and in the vacuum there acted an initial magnetic field parallel to the plane boundary of the half-space and there was no influence of coupling between temperature and strain fields.

Later, Massalas and Dalamangas [3] considered the same problem where the coupling between the temperature and strain fields was considered. Very recently Chatterjee and Roy Choudhuri [4] extended the problem [3] in generalized thermo-elasticity of Green and Lindsay taking into account the two relaxation times.

In the present paper we extend the problem [3] in generalized thermoelasticity by using the thermal relaxation time of Lord-Shulman theory [1]. The solutions for temperature distribution, deformation and perturbed magnetic field in the vacuum are obtained in analytical form in the first power of the magnetothermo-elastic coupling parameter ε and relaxation parameter τ_o'. In absence of ε, τ_o' the solutions agree with those in [2] and in absence of τ_o', the results agree with those in [3].

Surface stress for different times is calculated and graphically presented. It is believed that this particular problem has not been considered earlier.

2. PROBLEM FORMULATION.

We assume that a magneto-thermo-elastic wave is produced in an elastic half-space $x_1 > 0$ due to the thermal shock $\theta(o,t)=\theta_o H(t)$ applied on $x_1=0$ where θ_o is a constant and H(t) is the Heaviside function. We also assume that in both the media there is an initial magnetic field acting in the direction of x_3-axis. The simplified equations of slowly moving bodies in electrodynamics after linearization are the following:

$$\vec{\nabla}\times\vec{h} = \frac{4\pi}{c}\vec{j}$$
$$\vec{\nabla}\times\vec{E} = -\frac{\mu o}{c}\frac{\partial\vec{h}}{\partial t} \tag{2.1}$$
$$\vec{\nabla}.\vec{h}=0,\quad \vec{E} = -\frac{\mu_o}{c}(\dot{\vec{\mu}}\times\vec{H}_o)$$

where $\vec{E}$ denotes the electric field, $\vec{h}$ is the perturbation of the magnetic field, $\vec{H}_o$ is the initial constant magnetic field, $\vec{j}$ is the current density vector, $\vec{u}$ denotes the displacement vector, μ_o is the magnetic permeability, σ is the electric conductivity and c is the velocity of light. The displacement equation of motion in thermo-elasticity including the electromagnetic effect after linerization is,

$$\mu\nabla^2\vec{u} + (\lambda+\mu)\,\vec{\nabla}\,(\vec{\nabla}.\vec{u}) + \frac{\mu_o}{4\pi}[(\vec{\nabla}\times\vec{h})\times\vec{H}_o] - \gamma\vec{\nabla}\theta = \rho\,\ddot{\vec{u}}\,. \tag{2.2}$$

Also the modified form of Fourier's law of heat conduction taking into account the thermal relaxation time [1] is

$$\rho c_v(\dot{\theta} + \tau_o\ddot{\theta}) + \gamma T_o(\dot{\Delta} + \tau_o\ddot{\Delta}) = K\,\theta,_{ii},\quad (i=1,2,3) \tag{2.3}$$

where λ,μ are the Lame' constants, γ is equal to $(3\lambda+4\mu)\,\alpha_T$, α_T is the co-efficient of linear thermal expansion, θ is equal to $T-T_o$; T_o,T are the reference and absolute temperature of the body respectively; K is the co-efficient of heat conduction; ρ is the mass density; c_v is the specific heat at constant volume; τ_o is the relaxation time. The magneto-thermo-elastic wave propagated in the medium $x_1 > 0$ is assumed to depend on x_1 and time t.

For $\vec{H}_o = (0,0,H_3)$ equations (2.1) reduce to

$$\vec{E} = \frac{\mu_o H_3}{c}(0,\,\dot{u}_1,0),\quad \dot{\vec{h}} = -\frac{c}{\mu_o}(0,0,\frac{\partial E_2}{\partial x_1}),\quad \vec{j} = \frac{c}{4\pi}(0,\,-\frac{\partial h_3}{\partial x_1},\,0). \tag{2.4}$$

Equations (2.2) and (2.3) then lead to

$$(\lambda+2\mu + a_o^2\rho)\,\frac{\partial^2 u_1}{\partial x_1^2} - \gamma\frac{\partial\theta}{\partial x_1} = \rho\ddot{u}_1 \tag{2.5}$$

$$\rho c_\nu\left(\frac{\partial\theta}{\partial t} + \tau_o \frac{\partial^2\theta}{\partial t^2}\right) + \gamma T_o\left(\frac{\partial^2 u_1}{\partial x_1 \partial t} + \tau_o \frac{\partial^3 u_1}{\partial x \partial t^2}\right) = K \frac{\partial^2\theta}{\partial x_1^2} \tag{2.6}$$

where $a_o = \sqrt{\frac{\mu_o H_3^2}{4\pi\rho}}$ is the Alfven wave velocity. For convenience, we shall use the notations $u_1 = u$, $x_1 = x$.

In vacuum the system of equations of electrodynamics are

$$\left(\frac{\partial^2}{\partial x'^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) h_3^o = 0$$

$$\left(\frac{\partial^2}{\partial x'^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) E_2^o = 0 \tag{2.7}$$

where $x' = -x$.

The components T_{11} and T_{11}^o of Maxwell's stress tensor in elastic medium and in vacuum are

$$T_{11} = -\frac{\mu o}{4\pi} h_3 H_3 \text{ and } T_{11}^o = -\frac{1}{4\pi} h_3^o H_3 .$$

The normal mechanical and thermal stress is

$$\sigma_{11} = (\lambda + 2\mu)\frac{\partial u}{\partial x} - \gamma\theta .$$

The boundary conditions to be satisifed are

$$\sigma_{11} + T_{11} - T_{11}^o = 0, \quad x = x' = 0 \tag{2.8}$$

$$E_2 = E_2^o \quad , \; x = x' = 0 \tag{2.9}$$

$$\theta(o,t) = \theta_o H(t). \tag{2.10}$$

3. SOLUTION OF THE PROBLEM.

To find the solution of the problem we now introduce the following notations and non-dimensional variables

$$c_1^2 = \frac{\lambda+2\mu}{\rho},\; c_o^2 = a_o^2 + c_1^2,\; \xi = \frac{c_o x}{\kappa},\; \tau = \frac{c_o^2 t}{\kappa},$$

$$U = \frac{C_o(\lambda + 2\mu + a_o^2 \rho)}{\kappa\gamma T_o} u, \quad z = \frac{\theta}{T_o}, \quad \varepsilon = \frac{\gamma^2 T_o}{C_\varepsilon(\lambda+2\mu + a_o^2\rho)}, \quad \kappa = \frac{K}{\rho C_\nu},$$

$$\tau_o' = \tau_o \omega^*, \quad \omega^* = \frac{\rho C_\nu C_o^2}{K} = \frac{C_o^2}{\kappa}, \quad C_\varepsilon = \rho C_\nu .$$

The equations (2.5) - (2.7) and boundary conditions (2.8) - (2.10) become

$$\frac{\partial^2 U}{\partial \xi^2} - \frac{\partial Z}{\partial \xi} - \frac{\partial^2 U}{\partial \tau^2} = 0, \quad \xi > 0 \tag{3.1}$$

$$\frac{\partial^2 Z}{\partial \xi^2} - \frac{\partial Z}{\partial \xi} - \tau_o' \frac{\partial^2 Z}{\partial \tau^2} - \varepsilon \frac{\partial^2 U}{\partial \xi \partial \tau} - \varepsilon \tau_o' \frac{\partial^3 U}{\partial \xi \partial \tau^2} = 0, \quad \xi > 0 \tag{3.2}$$

$$\frac{\partial h_3^o}{\partial \xi'^2} - \beta^2 \frac{\partial^2 h_3^o}{\partial \tau^2} = 0, \quad \xi' > 0 \tag{3.3}$$

$$\frac{\partial U}{\partial \xi} - z + \beta_1 h_3^o = 0, \qquad \xi = \xi' = 0 \tag{3.4}$$

$$\beta_2 \frac{\partial^2 U}{\partial \tau^2} - \frac{\partial h_3}{\partial \xi'} = 0, \quad \xi = \xi' = 0, \tag{3.5}$$

$$Z(o, \tau) = \frac{\theta_o}{T_o} H(\tau), \tag{3.6}$$

where $\beta_1 = \dfrac{H_3}{4\pi\gamma T_o}, \quad \beta_2 = \dfrac{\mu_o H_3 \gamma T_o}{\rho c^2}, \quad \beta = \dfrac{C_o}{C}, \quad \xi' = -\xi.$

Initial conditions in the new variables are

$$U(\xi, o) = 0, \; Z(\xi, o) = 0, \; \frac{\partial z(\xi, o)}{\partial \xi} = 0.$$

We now introduce a potential function ϕ defined by

$$U = \frac{\partial \phi}{\partial \xi} . \tag{3.7}$$

Using (3.7) in (3.1) and then integrating we get

$$z(\xi, \tau) = \left(\frac{\partial^2}{\partial \xi^2} - \frac{\partial^2}{\partial \tau^2}\right) \phi \text{ in } \xi > 0 . \tag{3.8}$$

Using (3.7), the equation (3.2) leads to

$$\frac{\partial^2 z}{\partial \xi^2} - \frac{\partial z}{\partial \tau} - \tau_o' \frac{\partial^2 z}{\partial \tau^2} - \varepsilon \frac{\partial^3 \phi}{\partial \xi^2 \partial \tau} - \varepsilon \tau_o' \frac{\partial^4 \phi}{\partial \xi^2 \partial \tau^2} = 0 \quad (3.9)$$

In the Laplace transform domain the equations (3.8), (3.9) and (3.3) become

$$\bar{Z}(\xi,s) = (\frac{\partial^2}{\partial \xi^2} - s^2)\,\bar{\phi}, \quad \xi > 0 \quad (3.10)$$

$$(\frac{\partial^2}{\partial \xi^2} - s - \tau_o' s^2)\,\bar{z} = \varepsilon\, s(1+\tau_o' s)\,\frac{\partial^2 \bar{\phi}}{\partial \xi^2}, \quad \xi > 0 \quad (3.11)$$

$$\bar{h}_3{}^o = c_3\, e^{-\beta s \xi'}, \quad \xi > 0. \quad (3.12)$$

In Laplace transform domain, the boundary conditions (3.4) - (3.6) reduce to

$$\frac{\partial^2 \bar{\phi}}{\partial \xi^2} - \bar{z} + \beta_1 \bar{h}_3{}^o = 0, \quad \xi = 0 \quad (3.13)$$

$$\beta_2 s^2 \frac{\partial \bar{\phi}}{\partial \xi} - \frac{\partial \bar{h}_3{}^o}{\partial \xi'} = 0, \quad \xi = \xi' = 0 \quad (3.14)$$

$$\bar{Z}(o,s) = \frac{\theta_o}{T_o}\frac{1}{s}. \quad (3.15)$$

Eliminating $\bar{z}$ from (3.10) and (3.11) we get

$$\frac{\partial^4 \bar{\phi}}{\partial \xi^4} - \{1+\varepsilon+s+(1+\varepsilon)\,\tau_o' s\}s\,\frac{\partial^2 \bar{\phi}}{\partial \xi^2} + s^3(1+\tau_o' s)\,\bar{\phi} = 0. \quad (3.16)$$

The equation (3.16) reduces to (31) in [4] on setting $\alpha' = \alpha^{*\prime} = \tau_o'$.
The general solution of (3.16) vanishing at $\xi = \infty$ is

$$\bar{\phi}(\xi,s) = C_1 e^{-\lambda_1 \xi} + C_2\, e^{-\lambda_2 \xi}, \quad \xi > 0 \quad (3.17)$$

where λ_1, λ_2 are given by the roots of the equation

$$\lambda^4 - s\{1+\varepsilon+s+(1+\varepsilon)\,\tau_o' s\}\,\lambda^2 + s^3(1+\tau_o' s) = 0. \quad (3.18)$$

Hence

$$\lambda_{1,2} = [\tfrac{s}{2}\{s+1+\varepsilon+\tau_o'\varepsilon s+\tau_o' s) \pm [(1+\varepsilon^2\tau_o'^2 + \tau_o'^2 + 2\varepsilon\tau_o' + 2\varepsilon\tau_o'^2 - 2\tau_o')s^2$$

$$+ 2(\varepsilon - 1+2\varepsilon\tau_o' + \tau_o' + \varepsilon^2\tau_o')\, s + (1+\varepsilon)^2]^{1/2}\}]^{1/2}. \quad (3.19)$$

The equation (3.19) agrees with that of (34) in [4] for $\alpha' = \alpha^{*'} = \tau'_o$. For $\alpha' = \alpha^{*'} = 0$, the equations (3.16), (3.19) are in agreement with that of (24) in [3]. Thus the equations (3.1), (3.2), (3.16), (3.19) are more general in the sense that they incorporate the effect of thermal relaxation time of Lord-Shulman theory. From (3.10) using (3.17) we have

$$\bar{z}(\xi,s) = C_1(\lambda_1^2 - s^2)\, e^{-\lambda_1\xi} + C_2(\lambda_2^2 - s^2)\, e^{-\lambda_2\xi}, \quad \xi > 0. \tag{3.20}$$

From the boundary conditions (3.13) - (3.15) taking into account (3.17) and (3.20) we obtain a linear algebraic system with respect to C_1, C_2 and C_3 as

$$C_1 s^2 + C_2 s^2 + \beta_1 C_3 = 0, \quad \text{at } \xi = \xi' = 0 \tag{3.21}$$

$$\beta_2 s\lambda_1 C_1 + \beta_2 s\lambda_2 C_2 - \beta C_3 = 0, \text{ at } \xi = \xi' = 0 \tag{3.22}$$

$$C_1(\lambda_1^2 - s^2) + C_2(\lambda_2^2 - s^2) = \frac{\theta_o}{T_o s}. \tag{3.23}$$

The constants $C_i (i=1,2,3)$ being determined by (3.21) - (3.23), the solutions for $\bar{\phi}, \bar{Z}, \bar{U}, \bar{h}_3^{\,o}$ are given by

$$\bar{\phi}(\xi,s,\varepsilon,\tau'_o) = \frac{\theta_o}{T_o}\left[\frac{(s\beta+\beta_1\beta_2\lambda_2)\, e^{-\lambda_1\xi} - (s\beta + \beta_1\beta_2\lambda_1)\, e^{-\lambda_2\xi}}{s(\lambda_1 - \lambda_2)(\beta_1\beta_2 s^2 + \beta(\lambda_1+\lambda_2)s + \beta_1\beta_2\lambda_1\lambda_2}\right] \tag{3.24}$$

$$\bar{Z}(\xi,s,\varepsilon\tau'_o) = \frac{\theta_o}{T_o}\left[\frac{(\lambda_1^2-s^2)(s\beta+\beta_1\beta_2\lambda_2)\, e^{-\lambda_1\xi} - (\lambda_2^2-s^2)(s\beta+\beta_1\beta_2\lambda_1)\, e^{-\lambda_2\xi}}{s(\lambda_1-\lambda_2)(\beta_1\beta_2 s^2 + \beta(\lambda_1+\lambda_2)s + \beta_1\beta_2\lambda_1\lambda_2}\right] \tag{3.25}$$

$$\bar{U}(\xi,s,\varepsilon,\tau'_o) = \frac{\theta_o}{T_o}\left[\frac{\lambda_2(s\beta + \beta_1\beta_2\lambda_1)\, e^{-\lambda_2\xi} - \lambda_1(s\beta + \beta_1\beta_2\lambda_2)\, e^{-\lambda_1\xi}}{s(\lambda_1-\lambda_2)\,(\beta_1\beta_2 s^2 + \beta(\lambda_1+\lambda_2)s + \beta_1\beta_2\,\lambda_1\lambda_2}\right], \quad \xi > 0 \tag{3.26}$$

$$\bar{h}_3^{\,o}(\xi' s,\varepsilon,\tau'_o) = \frac{\theta_o}{T_o}\,\frac{s\beta_2\, e^{-\beta s\xi'}}{\beta_1\beta_2 s^2 + \beta(\lambda_1+\lambda_2)s + \beta_1\beta_2\lambda_1\lambda_2}, \quad \xi' > 0. \tag{3.27}$$

Since ε, $\tau'_o < 1$ for small thermo-elastic couplings, we expand the functions $\bar{z}$, $\bar{U}$, $\bar{h}_3^{\,o}$ into Maclaurian's series and retain the first two terms in the series expansion to obtain

$$\bar{Z}(\xi,s,\varepsilon,\tau_o') \simeq \frac{\theta_o}{T_o}\Big[\frac{e^{-\xi\sqrt{s}}}{s} + \varepsilon\Big\{\frac{\beta e^{-\xi s}}{(\beta+\beta_1\beta_2)(s-1)^2} + \frac{\beta_1\beta_2 e^{-\xi s}}{(\beta+\beta_1\beta_2)\sqrt{s}(s-1)^2} + \frac{e^{-\xi\sqrt{s}}}{s(s-1)^2}$$

$$+ \frac{\beta_1\beta_2}{2(\beta+\beta_1\beta_2)s(s-1)} + \frac{\xi}{2}\frac{e^{-\xi\sqrt{s}}}{\sqrt{s}(s-1)} - \frac{e^{-\xi\sqrt{s}}}{2s(\sqrt{s}-1)^2} - \frac{\beta e^{-\xi\sqrt{s}}}{2(\beta+\beta_1\beta_2)s(\sqrt{s}+1)^2}\Big\}$$

$$+ \tau_o'\Big\{-\frac{\xi\sqrt{s}\, e^{-\xi\sqrt{s}}}{2}\Big\}\Big] \tag{3.28}$$

$$\bar{U}(\xi,s,\varepsilon,\tau_o') \simeq \frac{\theta_o}{T_o}\Big[\frac{e^{-\xi\sqrt{s}}}{s\sqrt{s}(s-1)} - \frac{\beta e^{-\xi s}}{(\beta+\beta_1\beta_2)s(s-1)} - \frac{\beta\beta_2 e^{-\xi s}}{(\beta+\beta_1\beta_2)s\sqrt{s}(s-1)} + \varepsilon\Big\{-\frac{\beta e^{-\xi\sqrt{s}}}{2(\beta+\beta_1\beta_2)s\sqrt{s}(s-1)^2}$$

$$+ \frac{\xi\beta e^{-\xi\sqrt{s}}}{(\beta+\beta_1\beta_2)s(s-1)^2} - \frac{\beta e^{-\xi s}}{2(\beta+\beta_1\beta_2)s(s-1)^2} + \frac{\xi\beta e^{-\xi s}}{2(\beta+\beta_1\beta_2)(s-1)^2} + \frac{\xi\beta_1\beta_2 e^{-\xi\sqrt{s}}}{2(\beta+\beta_1\beta_2)s(s-1)^2} +$$

$$\frac{\xi\beta_1\beta_2 e^{-\xi s}}{2(\beta+\beta_1\beta_2)\sqrt{s}(s-1)^2} - \frac{e^{-\xi\sqrt{s}}}{2s\sqrt{s}(s-1)(\sqrt{s}-1)^2} + \frac{\beta e^{-\xi s}}{2(\beta+\beta_1\beta_2)s(s-1)(\sqrt{s}-1)^2}$$

$$+ \frac{\beta_1\beta_2 e^{-\xi s}}{2(\beta+\beta_1\beta_2)s\sqrt{s}(s-1)(\sqrt{s}-1)^2} + \frac{\beta^2 e^{-\xi s}}{2(\beta+\beta_1\beta_2)^2 s(s-1)(\sqrt{s}+1)^2} - \frac{\beta e^{-\xi\sqrt{s}}}{2(\beta+\beta_1\beta_2)s\sqrt{s}(s-1)(\sqrt{s}+1)^2}$$

$$+ \frac{\beta\beta_1\beta_2 e^{-\xi s}}{2(\beta+\beta_1\beta_2)^2 s\sqrt{s}(s-1)(\sqrt{s}+1)^2}\Big\} + \tau_o'\Big\{\frac{e^{-\xi\sqrt{s}}}{2\sqrt{s}(s-1)} - \frac{\xi e^{-\xi\sqrt{s}}}{2(s-1)} - \frac{\beta_1\beta_2 e^{-\xi s}}{2(\beta+\beta_1\beta_2)\sqrt{s}(s-1)}$$

$$- \frac{\beta e^{-\xi s}}{(\beta+\beta_1\beta_2)(s-1)^2} + \frac{\beta e^{-\xi s}}{(\beta+\beta_1\beta_2)\sqrt{s}(s-1)^2}\Big\}\Big] \tag{3.29}$$

$$\bar{h}_3^{\,o}(\xi',s,\varepsilon,\tau_o') \simeq \frac{\theta_o}{T_o}\Big[\frac{\beta_2 e^{-\beta s\xi'}}{(\beta+\beta_1\beta_2)\sqrt{s}(\sqrt{s}+1)} - \varepsilon\frac{\beta\beta_2 e^{-\beta s\xi'}}{2(\beta+\beta_1\beta_2)^2\sqrt{s}(\sqrt{s}+1)^3}$$

$$- \tau_o'\frac{\beta_2\sqrt{s}\, e^{-\beta s\xi'}}{2(\beta+\beta_1\beta_2)(\sqrt{s}+1)^2}\Big] \tag{3.30}$$

Taking inverse Laplace transform we obtain (Chatterjee (Roy) and Roy Choudhuri [4], Hetnarski [5], Oberhettiner and Badii [6]),

$$Z(\xi,\tau,\varepsilon,\tau_o') \simeq \frac{\theta_o}{T_o}\,[\mathrm{erfc}\,(\frac{\xi}{2\sqrt{\tau}}) + \varepsilon\,\{\frac{\beta}{\beta+\beta_1\beta_2}\,(\tau-\xi)\,e^{(\tau-\xi)}H(\tau-\xi) + \frac{\beta_1\beta_2}{\beta+\beta_1\beta_2}[\sqrt{\frac{\tau-\xi}{\pi}}$$

$$+ (\tau-\xi-\frac{1}{2})\,e^{(\tau-\xi)}\mathrm{erf}\,\sqrt{\tau-\xi}]\,H(\tau-\xi) + \tau f_1(\xi,\tau) - \frac{\xi}{2}f_2(\xi,\tau) - f_1(\xi,\tau) + \mathrm{erfc}(\frac{\xi}{2\sqrt{\tau}})$$

$$+ \frac{\beta_1\beta_2}{2(\beta+\beta_1\beta_2)}\,[f_1(\xi,\tau) - \mathrm{erfc}\,(\frac{\xi}{2\sqrt{\tau}})] + \frac{\xi}{2}f_2(\xi,\tau) - \frac{1}{2}f_3^{I}\,(\xi,\tau)$$

$$- \frac{\beta}{2(\beta+\beta_1\beta_2)}\,f_3^{II}\,(\xi,\tau)\} - \tau_o'\,\{\frac{\xi}{2}\,\frac{1}{4\sqrt{\pi}}\,(\xi^2 - 2\tau)\,\tau^{-5/2}e^{\frac{\xi^2}{4\tau}}\}] \qquad (3.31)$$

$$U(\xi,\tau,\varepsilon,\tau_o') \simeq \frac{\theta_o}{T_o}\,f_2(\xi,\tau) - 2\sqrt{\frac{\tau}{\pi}}\,e^{-(\frac{\xi^2}{4\tau})} + \xi\,\mathrm{erfc}\,(\frac{\xi}{2\sqrt{\tau}}) - \frac{\beta}{\beta+\beta_1\beta_2}\,(e^{\tau-\xi}-1)H(\tau-\xi)$$

$$- \frac{\beta_1\beta_2}{\beta+\beta_1\beta_2}\,[e^{(\tau-\xi)}\,\mathrm{erf}\,\sqrt{\tau-\xi} - 2\sqrt{\frac{\tau-\xi}{\pi}}]\,H(\tau-\xi) + \varepsilon\{-\frac{\beta}{2(\beta+\beta_1\beta_2)}\,[f_3(\xi,\tau) - f_2(\xi,\tau)$$

$$+ 2\sqrt{\frac{\tau}{\Pi}}\,e^{-(\frac{\xi^2}{4\tau})} - \xi\,\mathrm{erfc}\,(\frac{\xi}{2\sqrt{\tau}})] + \frac{\xi\beta}{2(\beta+\beta_1\beta_2)}\,[\tau f_1(\xi,\tau) - \frac{\xi}{2}f_2(\xi,\tau) - f_1(\xi,\tau) + \mathrm{erfc}(\frac{\xi}{2\sqrt{\tau}})]$$

$$- \frac{\beta}{2(\beta+\beta_1\beta_2)}(\tau-\xi-1)\,e^{(\tau-\xi)}H(\tau-\xi) + \frac{\xi\beta}{2(\beta+\beta_1\beta_2)}(\tau-\xi)e^{(\tau-\xi)}H(\tau-\xi) + \frac{\xi\beta_1\beta_2}{2(\beta+\beta_1\beta_2)}[\tau f_1(\xi,\tau)$$

$$-(\xi/2)f_2(\xi,\tau) - f_1(\xi,\tau) + \mathrm{erfc}(\frac{\xi}{2\sqrt{\tau}})] + \frac{\xi\beta_1\beta_2}{2(\beta+\beta_1\beta_2)}[\frac{\sqrt{\tau-\xi}}{\pi} + (\tau-\xi-\frac{1}{2})\,e^{(\tau-\xi)}\mathrm{erfc}\sqrt{\tau-\xi}]\,H(\tau-\xi)$$

$$+ f_4(\xi,\tau) + 4\sqrt{\frac{\tau}{\pi}}\,e^{-(\frac{\xi^2}{4\tau})} - \mathrm{erfc}(\frac{\xi}{2\sqrt{\tau}})(\tau-\xi) - (2\tau-\xi-2)e^{(\tau-\xi)}\mathrm{erfc}(\frac{\xi}{2\sqrt{\tau}} - \sqrt{\tau})$$

$$+ \frac{\beta}{2(\beta+\beta_1\beta_2)}\,[(\tau-\xi-\frac{3}{2})\,\frac{\sqrt{\tau-3}}{\pi} - \frac{1}{2}\,(5\tau-5\xi-3)e^{(\tau-\xi)} - \frac{5}{2}\,(\tau-\xi-\frac{1}{2})e^{(\tau-\xi)}\mathrm{erf}\sqrt{\tau-\xi}$$

$$+ (\frac{3}{2}\,(\tau-\xi) + (\tau-\xi)^2)\,\mathrm{erf}\sqrt{\tau-\xi} + \frac{3}{2}\,(\tau-\xi) + (\tau-\xi)^2-1]\,H(\tau-\xi) + \frac{\beta_1\beta_2}{2(\beta+\beta_1\beta_2)}[(\tau-\xi-\frac{9}{4})\,\sqrt{\frac{\tau-\xi}{\pi}}$$

$$- \frac{1}{2}\,(7\tau-7\xi-5)e^{(\tau-\xi)} - \frac{1}{2}\,(7\tau-7\xi-\frac{11}{2})e^{(\tau-\xi)}\mathrm{erf}\sqrt{\tau-\xi} + (\frac{5}{2}(\tau-\xi)+(\tau-\xi)^2)\mathrm{erf}\sqrt{\tau-\xi}$$

$$+ \frac{5}{2}(\tau-\xi)+(\tau-\xi)^2-2]H(\tau-\xi) + \frac{\beta^2}{2(\beta+\beta_1\beta_2)^2}[-(\tau-\xi-\frac{3}{2})\sqrt{\frac{\tau-\xi}{\pi}} - \frac{1}{2}(5\tau-5\xi-3)e^{(\tau-\xi)}$$

$$+ \frac{5}{2}(\tau-\xi-\frac{1}{2})e^{(\tau-\xi)}\mathrm{erf}\sqrt{\tau-\xi} - (\frac{3}{2}(\tau-\xi) + (\tau-\xi)^2)\mathrm{erf}\sqrt{\tau-\xi} + \frac{3}{2}(\tau-\xi) + (\tau-\xi)^2-1] \quad H(\tau-\xi)$$

$$- \frac{\beta}{2(\beta+\beta_1\beta_2)}[f_5(\xi,\tau) - 4\sqrt{\frac{\tau}{\pi}}\, e^{-(\frac{\xi^2}{4\tau})} - \mathrm{erfc}(\frac{\xi}{2\sqrt{\tau}})(2+\xi)-(2\tau+\xi-2)e^{(\tau+\xi)}\mathrm{erfc}(\frac{\xi}{2\sqrt{\tau}}+\sqrt{\tau})]$$

$$+ \frac{\beta\beta_1\beta_2}{2(\beta+\beta_1\beta_2)^2}[(\tau-\xi-\frac{9}{4})\sqrt{\frac{\tau-\xi}{\pi}} + \frac{1}{2}(7\tau-7\xi-5)e^{(\tau-\xi)} - \frac{1}{2}(7\tau-7\xi-\frac{11}{2})e^{(\tau-\xi)}\mathrm{erf}\sqrt{\tau-\xi}$$

$$+ (\frac{5}{2}(\tau-\xi) + (\tau-\xi)^2)\mathrm{erf}\sqrt{\tau-\xi} - \frac{5}{2}(\tau-\xi)-(\tau-\xi)^2+2]H(\tau-\xi)\} + \; + \tau_o' \{\frac{1}{2} f_2(\xi,\tau) - \frac{\xi}{2} f_1(\xi,\tau)$$

$$- \frac{\beta_1\beta_2}{2(\beta+\beta_1\beta_2)} e^{(\tau-\xi)}\mathrm{erf}\sqrt{\tau-\xi}\, H(\tau-\xi) - \frac{\beta}{\beta+\beta_1\beta_2}(\tau-\xi)e^{(\tau-\xi)}H(\tau-\xi) +$$

$$\frac{\beta}{\beta(\beta+\beta_1\beta_2}[((\tau-\xi) - \frac{1}{2})e^{(\tau-\xi)}\mathrm{erf}\sqrt{\tau-\xi} + \sqrt{\frac{\tau-\xi}{\pi}}]\; H(\tau-\xi)\}]. \tag{3.32}$$

$$h_3^o(\xi',\tau,\varepsilon,\tau_o') \simeq \frac{\theta_o}{T_o}[\frac{\beta_2}{\beta+\beta_1\beta_2} e^{(\tau-\beta\xi')}\mathrm{erfc}\sqrt{\tau-\beta\xi'}\; H(\tau-\beta\xi') - \varepsilon\{-\frac{\beta\beta_2}{2(\beta+\beta_1\beta_2)^2} 2(\tau-\beta\xi')\sqrt{\frac{\tau-\beta\xi'}{\pi}}$$

$$+ [1-2(\tau-\beta\xi')^2]\; e^{(\tau-\beta\xi')}\mathrm{erfc}\sqrt{\tau-\beta\xi}\; H(\tau-\beta\xi')\} + \tau_o' \{-\frac{\beta_2}{2(\beta+\beta_1\beta_2)}[\frac{1}{\sqrt{\pi(\tau-\beta\xi')}}$$

$$- 2(\tau-\beta\xi')\; e^{(\tau-\beta\xi')}\mathrm{erfc}\sqrt{\tau-\beta\xi'} + 2\sqrt{\frac{\tau-\beta\xi'}{\pi}}]\; H(\tau-\beta\xi')\}]. \tag{3.33}$$

where the functions $f_i(\xi,\tau)$, $i=1,2,3,4,5$ are given by

$$f_1(\xi,\tau) = \frac{e^{\tau}}{2}[e^{-\xi}\mathrm{erfc}(\frac{\xi}{2\sqrt{\tau}} - \sqrt{\tau}) + e^{\xi}\mathrm{erfc}(\frac{\xi}{2\sqrt{\tau}} + \sqrt{\tau})]$$

$$f_2(\xi,\tau) = \frac{e^{\tau}}{2}[e^{-\xi}\mathrm{erfc}(\frac{\tau}{2\sqrt{\tau}} - \sqrt{\tau}) - e^{\xi}\,\mathrm{erfc}(\frac{\xi}{2\sqrt{\tau}} + \sqrt{\tau})]$$

$$f_3^{I}(\xi,\tau) == \mathrm{erfc}(\frac{\xi}{2\sqrt{\tau}}) + 2\sqrt{\frac{\tau}{\pi}}\; e^{-(\frac{\xi^2}{4\tau})} + (2\tau-\xi-1)e^{(\tau-\xi)}\mathrm{erfc}(\frac{\xi}{2\sqrt{\tau}} - \sqrt{\tau})$$

$$f_3^{II}(\xi,\tau) = \mathrm{erfc}(\frac{\xi}{2\sqrt{\tau}}) - 2\sqrt{\frac{\tau}{\pi}}\, e^{-(\frac{\xi^2}{4\tau})} + (2\tau+\xi-1)\; e^{(\tau+\xi)}\mathrm{erfc}(\frac{\xi}{2\sqrt{\tau}} + \sqrt{\tau})$$

$$f_4(\xi,\tau) = \int_0^{\tau} e^{m}\{2\sqrt{\frac{\tau-m}{\pi}}\; e^{-\frac{\xi^2}{4(\tau-m)}} + [2(\tau-m)-\xi]\; e^{(\tau-\xi-m)}\mathrm{erfc}(\frac{\xi}{2\sqrt{\tau-m}} - \sqrt{\tau-m})\}dm$$

$$f_5(\xi,\tau) = \int_0^{\tau} e^{m}\;\{2\sqrt{\frac{\tau-m}{\pi}}\; e^{-\frac{\xi^2}{4(\tau-m)}} - [2(\tau-m)+\xi]\; e^{(\tau+\xi-m)}\mathrm{erfc}(\frac{\xi}{2\sqrt{\tau-m}} + \sqrt{\tau-m})\}dm$$

where erfx and erfcx denote the error function and complementary error function respectively.

4. NUMERICAL RESULT.

The surface stress is given by

$$-\frac{T_{11}^{o}}{\frac{\theta_o H_3}{4\pi T_o}\;\frac{\beta_2}{\beta(1+\beta_3)}} = e^{\tau}(1-\mathrm{erf}\sqrt{\tau} - \frac{\varepsilon}{2(1+\beta_3)}\{-\,2\tau\sqrt{\frac{\tau}{\pi}} + (1-2\tau^2)e^{\tau}(1-\mathrm{erf}\sqrt{\tau})\}$$

$$-\tau_o'\{\frac{1}{2\sqrt{\pi\tau}} - \tau e^{\tau}(1-\mathrm{erf}\sqrt{\tau}) + \sqrt{\frac{\tau}{\pi}}\}$$

where $\beta_3 = \frac{\beta_1\beta_2}{\beta}$.

If there is no coupling between the electromagnetic field and strain field, $H_3 = 0$, $\beta_2 = 0$, $\beta_3 \to 0$ and β is finite so that $T_{11}^{o} = 0$ on $\xi = a_0$.
In presence of the electomagnetic field and strain field, the surface stress is given by

$$-\frac{T_{11}^{o}}{\frac{\theta_o H_3}{4\pi To}\;\frac{\beta_2}{\beta(1+\beta_3)}} = X(\tau,\varepsilon,\tau_o')$$

where

$$X(\tau,\varepsilon,\tau_o') = e^{\tau}(1 - \mathrm{erf}\sqrt{\tau}) - \frac{\varepsilon}{2(1+\beta_3)} \{- 2\tau\sqrt{\frac{\tau}{\pi}} + (1-2\tau^2)e^{\tau}(1-\mathrm{erf}\sqrt{\tau})\}$$

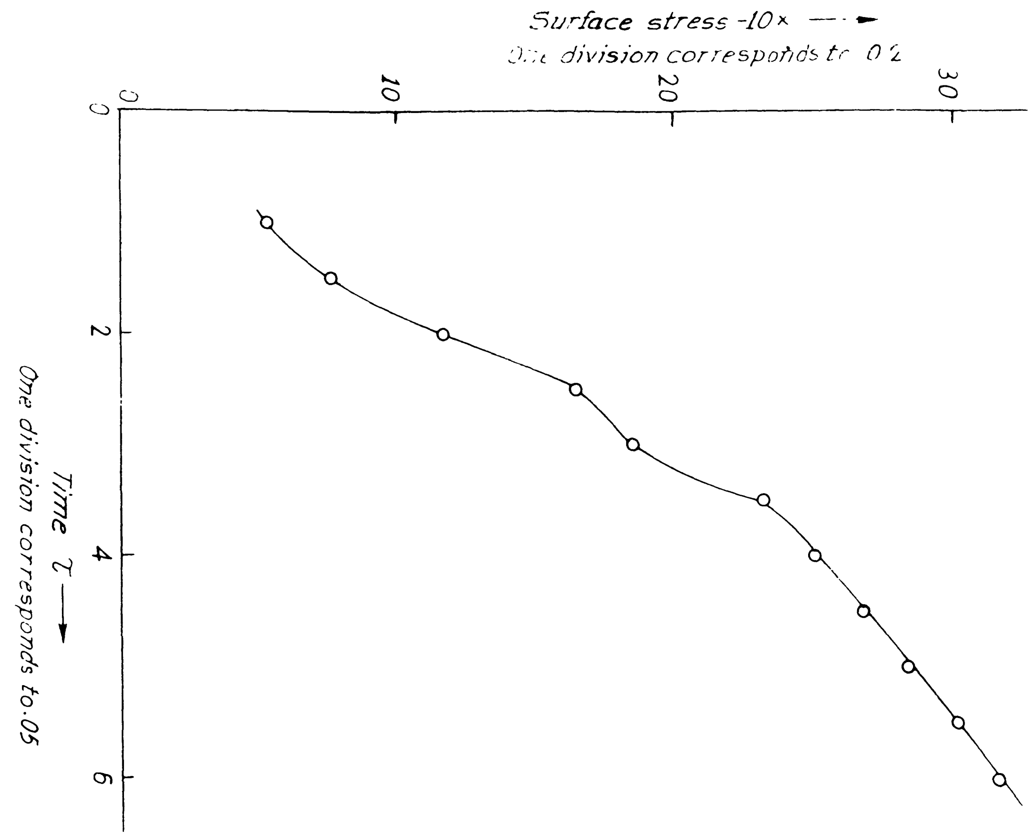
$$- \tau_o'\{\frac{1}{2\sqrt{\tau\pi}} - \tau\, e^{\tau}(1-\mathrm{erf}\sqrt{\tau}) + \sqrt{\frac{\tau}{\pi}}\}$$

We can assume $\beta_3 << 1$ since $c >> 1$ and a_o and C_o are finite. We take $\beta_3 = .05$. For numerical calculation we take the material of the half-space to be copper for which $\varepsilon = 0.0168$. If we assume that a representative value of the relaxation time τ_o is 10^{-11} (see [7]), then the non-dimensional thermal wave speed in copper should be approximately equal to 0.66.

Then $\tau_o' \simeq 2.3$ (For thermal properties and sound wave speed in copper, see ref. [8]).

Surface stress X for various values of times τ are exhibited in the following table and also graphically represented.

τ	-10X
1	5.4
1.5	7.7
2.0	11.8
2.5	16.5
3.5	23.2
4.0	25.0
4.5	26.8
5.0	28.3
5.5	30.0
6.0	31.5



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