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Research Article

Finding Global Minima with a Filled Function Approach for Non-Smooth Global Optimization

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A filled function approach is proposed for solving a non-smooth unconstrained global optimization problem. First, the definition of filled function in Zhang (2009) for smooth global optimization is extended to non-smooth case and a new one is put forwarded. Then, a novel filled function is proposed for non-smooth the global optimization and a corresponding non-smooth algorithm based on the filled function is designed. At last, a numerical test is made. The computational results demonstrate that the proposed approach is effcient and reliable.

1. Introduction

Because of advances in science, economics and engineering, studies on global optimization for multi-minimum nonlinear programming problem (P') : $\min_{x\in R^n} f(x)$ have become a topic of great concern. There are two difficulties faced by global optimization, one is how to leave the current solution for a better one, another is how to decide the current solution is a global one. So far, most existing methods deal only with the first issue. Among these methods, the filled function method is a practical useful tool for global optimization. It was first put forwarded by [1] for smooth unconstrained global optimization. The idea behind the filled function methods is to construct an auxiliary function that allows us to escape from a given local minimum of the original objective function. It consists of two phase: local minimization and filling. The two phases are used alternately until a global minimizer of (P') is found. The method has been further developed by [2–8]. In practical problems, however, objective functions are not always smooth, so several scholars have extended the filled function method for smooth global optimization to non-smooth cases(see [9]). In this paper, we modify the concept of filled function presented by [10] and propose a novel class of filled function for non-smooth the global optimization. This paper is divided into 6 sections. The next section presents some non-smooth preliminaries. In Section 3, the modified concept of the filled function for non-smooth global optimization is introduced, a novel class of filled function is given and its properties are investigated. In Section 4, a filled function algorithm is proposed. Section 5 presents some encouraging numerical results. Last, in Section 6, the conclusion is given.

2. Non-Smooth Preliminaries

To introduce the concept of the filled function approach for non-smooth global optimization, we recall some definitions and lemmas on non-smooth optimization which would be used in the next section.

Definition 2.1. Let X be a subset of R^n. A function $f : X \to R$ is said to be Lipschitz continuous with a constant L on X provided that, for some scalar $L > 0$, one has

$$|f(x) - f(y)| \leq L\|x - y\| \tag{2.1}$$

for all points $x, y \in X$.

Definition 2.2 (see [11]). Let f be Lipschitz with constant L at the point x, the generalized gradient of f at x is defined as

$$\partial f(x) = \left\{\xi \in R^n :< \xi,\ d \geqslant< f^0(x; d),\ \forall d \in X\right\}, \tag{2.2}$$

where $f^0(x; d) = \limsup_{y \to x, t \downarrow 0}((f(y + td) - f(y))/t)$ is the generalized directional derivative of $f(x)$ in the direction d at x.

Lemma 2.3 (see [11]). *Let f be Lipschitz with constant L at the point x, then*

(a) *$f^0(x; d)$ is finite, sublinear and satisfies*

$$\left|f^0(x; d)\right| \leq L\|d\|. \tag{2.3}$$

(b) *As a function of (x, d), $f^0(x; d)$ is super-semicontinuous; as a function of d, it is Lipschitz with constant L.*

(c) $\partial \Sigma s_i f_i(x) \subseteq \Sigma s_i \partial f_i(x)$, *for* $\forall s_i \in R$.

(d) *$\partial f(x)$ is a nonempty compact convex set, and to any $\xi \in \partial f(x)$, one has $\|\xi\| \leq L$.*

(e) $\forall d \in X$, $f^0(x; d) = \max\{< \xi, d >: \xi \in \partial f(x)\}$.

Lemma 2.4 (see [11]). *If x^* is a local minimizer of $f(x)$, then $0 \in \partial f(x^*)$.*

3. A New Filled Function and Its Properties

Consider problem(P'). To begin with, this paper makes the following assumptions.

Assumption 3.1. $f(x)$ is Lipschitz continuous with a constant L on R^n.

Assumption 3.2. $f(x)$ is coercive, that is, $f(x) \to +\infty$ as $\|x\| \to +\infty$.

Note that Assumption 3.2 implies the existence of a compact set X whose interior contains all minimizers of $f(x)$. We assume that the value of $f(x)$ for x located on the boundary of X is greater than the value of $f(x)$ for any x inside X. Then the original problem is equivalent to problem $(P) : \min_{x \in X} f(x)$.

Assumption 3.3. $f(x)$ has only a finite number of different minimal function values in X.

Let x^* be a local minimizer of (P). In [10], the filled function for smooth global optimization was defined as follows.

Definition 3.4. A function $P(x, x^*)$ is called a filled function of $f(x)$ at a local minimizer x^*, if $P(x, x^*)$ has the following properties:

(1) x^* is a strict maximizer of $P(x, x^*)$.

(2) $P(x, x^*)$ has no stationary points in the region $S_1 = \{x \in X \setminus \{x^*\} : f(x) \geq f(x^*)\}$.

(3) If x^* is not a global minimizer of (P), then $P(x, x^*)$ has at least one minimizer in the region $S_2 = \{x \in X : f(x) < f(x^*)\}$.

This paper extends about definition to non-smooth case and gives the following definition of a filled function.

Definition 3.5. A function $P(x, x^*)$ is called a filled function of $f(x)$ at a local minimizer x^*, if $P(x, x^*)$ has the following properties:

(1) x^* is a strict maximizer of $P(x, x^*)$.

(2) One has $0 \notin \partial P(x, x^*)$, for any $x \in S_1 = \{x \in X : f(x) \geq f(x^*), x \neq x^*\}$.

(3) If x^* is not a global minimizer of (P), then $P(x, x^*)$ has at least one minimizer in the region $S_2 = \{x \in X : f(x) < f(x^*)\}$.

For convenience, we use $L(P)$ and $G(P)$ to denote the set of local minimizers and the set of global minimizers of problem (P), respectively.

In what follows, we first design a function $\varphi(t)$ satisfying the following conditions:

(1) $\varphi(0) = 0$,

(2) $\forall t \in [-t_1, \infty), \varphi'(t) > 0$ (where $t_1 \geq 0$),

(3) $\lim_{t \to +\infty}(t\varphi'(t)/\varphi(t)) = 0$,

(4) $\varphi''(t) \leq 0$ for any $t \geq 0$.

Some examples of the function $\varphi(t)$ with the properties 1–4 are $\ln(1+t)$, $t/(1+t)$, $1 - \exp(-t)$.

Now, a filled function with two parameters for non-smooth global optimization is constructed as follows

$$F(x, x^*, q, r) = \frac{1}{1 + q\|x - x^*\|}\varphi\left(q\left|f(x) - f(x^*) + r\right|\right), \tag{3.1}$$

where $q > 0$ and $r > 0$ are parameters, r satisfies $0 < r < f(x^*) - f(x_G)$, where $x_G \in G(P)$.

Next, we will show that the function $F(x, x^*, q, r)$ is a filled function satisfying Definition 3.5.

Theorem 3.6. *Let $x^* \in L(P)$. If $q > 0$ is large enough such that $L > (\varphi'(qr)/\varphi(qr))$, then x^* is a strict maximizer of $F(x, x^*, q, r)$.*

Proof. Since $x^* \in L(P)$, there exists a neighborhood $O(x^*, \delta)$ of x^* with $\delta > 0$ such that $f(x) \geq f(x^*)$ for all $x \in O(x^*, \delta) \bigcap X$ and $x \neq x^*$. By the mean value theorem, it follows that

$$\begin{aligned} F(x, x^*, q, r) &= \frac{\varphi(q(f(x) - f(x^*) + r))}{1 + q\|x - x^*\|} \leq \frac{\varphi[q(L\|x - x^*\| + r)]}{1 + q\|x - x^*\|} \\ &= \frac{\varphi(qr) + qL\|x - x^*\|\varphi'[qr + \theta qL\|x - x^*\|]}{1 + q\|x - x^*\|} \\ &\leq \frac{\varphi(qr) + qL\|x - x^*\|\varphi'(qr)}{1 + q\|x - x^*\|}, \end{aligned} \tag{3.2}$$

where $\theta \in (0, 1)$.

By the property (3) of $\varphi(t)$, when q is sufficiently large such that

$$q > qr\frac{\varphi'(qr)}{\varphi(qr)}\frac{L}{r}, \tag{3.3}$$

we have that

$$\frac{\varphi(qr) + qL\|x - x^*\|\varphi'(qr)}{1 + q\|x - x^*\|} < \frac{\varphi(qr) + q\|x - x^*\|\varphi(qr)}{1 + q\|x - x^*\|} = \varphi(qr) = F(x^*, x^*, q, r). \tag{3.4}$$

Therefore we obtain that

$$F(x, x^*, q, r) < F(x^*, x^*, q, r) \quad \text{for any } x \in O(x^*, \delta) \bigcap X \text{ with } x \neq x^*. \tag{3.5}$$

Hence, x^* is a strict maximizer of $F(x, x^*, q, r)$. □

Theorem 3.7. *Assume that $x^* \in L(P)$. To any $x \in S_1$, if $q > 0$ is large enough such that $qL(1 + M)(\varphi'(qr)/\varphi(qr)) < 1$, where $M = \max_{x \in X}\|x - x^*\|$, then one has $0 \notin \partial F(x, x^*, q, r)$. In other words, x is not a stationary point of $F(x, x^*, q, r)$.*

Proof. We first note that for any $x \in S_1$, one has $f(x) \geq f(x^*)$, $x \neq x^*$, and

$$\partial F(x, x^*, q, r) \subset \frac{q\varphi'(q(f(x) - f(x^*) + r))}{1 + q\|x - x^*\|}\partial f(x) - \frac{q\varphi(q(f(x) - f(x^*) + r))}{(1 + q\|x - x^*\|)^2}\frac{x - x^*}{\|x - x^*\|}. \tag{3.6}$$

Denoting $d = (x - x^*)/\|x - x^*\|$, for any $\xi \in \partial F(x, x^*, q, r)$, there exists $\eta \in \partial f(x)$ such that

$$\begin{aligned}
\langle \xi, d \rangle &= \left\langle \frac{q\varphi'(q(f(x) - f(x^*) + r))}{1 + q\|x - x^*\|}\eta - \frac{q\varphi(q(f(x) - f(x^*) + r))}{(1 + q\|x - x^*\|)^2} \frac{x - x^*}{\|x - x^*\|}, \frac{x - x^*}{\|x - x^*\|} \right\rangle \\
&= \frac{q\varphi(q(f(x)-f(x^*)+r))}{(1+q\|x-x^*\|)^2} \times \left[\frac{\varphi'(q(f(x)-f(x^*)+r))}{\varphi(q(f(x)-f(x^*)+r))} \times \frac{1+q\|x-x^*\|}{\|x-x^*\|} \times (x-x^*)^T\eta - 1 \right] \\
&\leq \frac{q\varphi(q(f(x) - f(x^*) + r))}{(1 + q\|x - x^*\|)^2} \left[\frac{\varphi'(qr)}{\varphi(qr)} (1 + qM)Ł - 1 \right] \\
&\leq \frac{q\varphi(q(f(x) - f(x^*) + r))}{(1 + q\|x - x^*\|)^2} \left[Lqr \frac{\varphi'(qr)}{\varphi(qr)} \frac{(1 + M)}{r - 1} \right] < 0.
\end{aligned} \tag{3.7}$$

So, to any $\xi \in \partial F(x, x^*, q, r)$, one has $\xi^T d < 0$. Then $0 \notin \partial F(x, x^*, q, r)$. □

Theorem 3.8. *Assume that $x^* \in L(P) \setminus G(P)$. Then there exists a point $x_0^* \in S_2 = \{x | f(x) < f(x^*), x \in X\}$ such that x_0^* is a minimizer of $F(x, x^*, q, r)$.*

Proof. Since $x^* \in L(P) \setminus G(P)$, then there exists a point $x^{**} \in G(P)$ such that $f(x^{**}) < f(x^*)$. Now, by the choice of parameter of r, one has

$$f(x^{**}) - f(x^*) + r < 0, \tag{3.8}$$

so that there exists at least one point $x_0^* \in X$, such that

$$f(x_0^*) - f(x^*) + r = 0. \tag{3.9}$$

It follows that $F(x_0^*, x^*, q, r) = 0$. On the other hand, by the definition of $F(x, x^*, q, r)$, we have $F(x, x^*, q, r) \geq 0$. Therefore, we conclude $F(x, x^*, q, r) \geq F(x_0^*, x^*, q, r)$ for all $x \in X$, which implies that x_0^* is a minimizer of $F(x, x^*, q, r)$.

Theorem 3.6–3.3 state clearly that the proposed filled function satisfies the properties 1–3 of Definition 3.5. □

Theorem 3.9. *Suppose that $x_1, x_2 \in S_1$ and $\|x_1 - x^*\| > \|x_2 - x^*\| > 0$.*

(a) *If there exists a constant $B > 0$ such that $\lim_{t\to+\infty}\varphi(t) = B$, then, for sufficiently large $q > 0$, one has $F(x_1, x^*, q, r) < F(x_2, x^*, q, r)$.*

(b) *If there exists a constant $C > 0$ such that $\lim_{t\to+\infty}(\varphi(t)/\ln(1+t)) = C$, then for sufficiently large $q > 0$, it holds $F(x_1, x^*, q, r) < F(x_2, x^*, q, r)$.*

Proof. Let $x_1, x_2 \in S_1$, that is $f(x_1) \geq f(x^*), f(x_2) \geq f(x^*)$. For simplicity, let $\hat{f}_1 = f(x_1) - f(x^*) + r, \hat{f}_2 = f(x_2) - f(x^*) + r$.

(a) In this case, we can see that

$$\lim_{q \to +\infty} \frac{\varphi\left(q\hat{f}_2\right)}{\varphi\left(q\hat{f}_1\right)} = 1 ,$$
$$\lim_{q \to +\infty} \frac{1 + q\|x_2 - x^*\|}{1 + q\|x_1 - x^*\|} = \frac{\|x_2 - x^*\|}{\|x_1 - x^*\|} < 1, \tag{3.10}$$

since $\lim_{t \to +\infty} \varphi(t) = B$ and $\|x_1 - x^*\| > \|x_2 - x^*\| > 0$.

Therefore, for large q, there exists

$$\frac{\varphi\left(q\hat{f}_2\right)}{\varphi\left(q\hat{f}_1\right)} > \frac{1 + q\|x_2 - x^*\|}{1 + q\|x_1 - x^*\|}. \tag{3.11}$$

It follows that $F(x_1, x^*, q, r) < F(x_2, x^*, q, r)$.

(b) If $\varphi(t) = \ln(1 + t)$ and $q > 0$ is sufficiently large, then

$$\frac{\ln\left(1 + q\hat{f}_2\right)}{\ln\left(1 + q\hat{f}_1\right)} > \frac{1 + q\|x_2 - x^*\|}{1 + q\|x_1 - x^*\|}. \tag{3.12}$$

Thus, we have $F(x_1, x^*, q, r) < F(x_2, x^*, q, r)$.

If $\varphi(t) \neq \ln(1 + t)$ but $\lim_{t \to +\infty}(\varphi(t)/\ln(1 + t)) = C$, then

$$\lim_{q \to +\infty} \frac{\varphi\left(q\hat{f}_2\right)}{\varphi\left(q\hat{f}_1\right)} = \lim_{q \to +\infty} \left[\frac{\varphi\left(q\hat{f}_2\right)}{\ln\left(1 + q\hat{f}_2\right)} \cdot \frac{\ln\left(1 + q\hat{f}_1\right)}{\varphi\left(q\hat{f}_1\right)} \cdot \frac{\ln\left(1 + q\hat{f}_2\right)}{\ln\left(1 + q\hat{f}_1\right)}\right] = 1,$$
$$\frac{\varphi\left(q\hat{f}_2\right)}{\varphi\left(q\hat{f}_1\right)} > \frac{1 + q\|x_2 - x^*\|}{1 + q\|x_1 - x^*\|}. \tag{3.13}$$

Therefore, $F(x_1, x^*, q, r) < F(x_2, x^*, q, r)$. □

4. Solution Algorithm

In this section, we state our algorithm(NFFA)] for non-smooth global optimization based on the previous proposed filled function.

Algorithm NFFA

Initialization Step:

(1) Set a disturbance $\delta = 0.1$.

(2) Choose an upper bound $q_U > 0$ of q, for example, set $q_U := 10^8$.

(3) Set $\hat{q} = 10$.

(4) Choose directions $e_k, k = 1, 2, \ldots, k_0$, where $k_0 \geq 2n$, n is the number of variables.

(5) Specify an initial point $x \in X$ to start phase 1 of the algorithm.

(6) Set $r = 10^{-6}$.

(7) Set $k := 1$.

Main Step

(1) Starting from $x \in X$, activate a non-smooth local minimization procedure to minimize $f(x)$, and find its local minimizer x_1^*.

(2) Let $q = 1$.

(3) Construct the filled function as follows:

$$F(x, x_1^*, q, r) = \frac{1}{1 + q\|x - x_1^*\|}\varphi\left(q\left|f(x) - f(x_1^*) + r\right|\right). \tag{4.1}$$

(4) **else If** $k > k_0$, **then** go to 6.

Use $x := x_1^* + \delta e_k$ as an initial point, minimize the filled function problem(FP) : $\min_{x \in X} F(x, x_1^*, q, r)$ by implementing a non-smooth local minimization procedure and obtain a local minimizer x_k.

(5) **If** x_k satisfies $f(x_k) < f(x_1^*)$, **then** set $x := x_k$ and $k := 1$. Use point x as a new initial point, minimize problem (P) by implementing a local search procedure and obtain another local minimizer x_2^* of $f(x)$ such that $f(x_2^*) < f(x_1^*)$, set $x_1^* := x_2^*$, go to 2; Otherwise, set $k := k + 1$, go to 4.

(6) Increase q by setting $q := \hat{q}q$.

(7) **If** $q \leq q_U$, **then** set $k := 1$, go to 3; else the algorithm is incapable of finding a better local minimizer, the algorithm stops and x_1^* is taken as a global minimizer.

The motivation and mechanism behind this algorithm are explained below.

In Step 4 of the Initialization step, we choose direction $e_k, k = 1, 2, \ldots, k_0$ as positive and negative unit coordinate vectors, where $k_0 = 2n$. For example, when $n = 2$, the directions can be chosen as $(1, 0), (0, 1), (-1, 0), (0, -1)$.

In Steps 1, 4 and 5 of the Main step, we minimize problem (P) by applying non-smooth local optimization algorithms, such as Hybrid Hooke and Jeeves-Direct Method for Non-smooth Optimization[12], Mesh Adaptive Direct Search Algorithms for Constrained Optimization [13], Bundle methods, Powell's method, and so forth. In particular, the Hybrid Hooke and Jeeves-Direct Method is more preferable to others, since it is guaranteed to find a local minimum of a non-smooth function subject to simple bounds.

Recall from Theorems 3.7 and 3.8 that the value of q should be selected sufficiently large. In Main Step 2, we first set $q = 1$, then it is gradually increased until it reaches the preset upper bound q_U. If the parameter q exceeds q_U and we cannot find a point $x \in X$ such that $f(x) < f(x_1^*)$, then we believe that there does not exist a better local minimizer of problem (P), the current local minimizer is taken as a global minimizer and the algorithm is terminated.

5. Numerical Experiment

In this section, we apply the above algorithm to several test problems to demonstrate its efficiency. All the numerical experiments are implemented in Fortran 95, under Windows XP and Pentium (R) 4 CPU 2.80 GMHZ. In our programs, the filled function is of the form

$$F(x, x^*, q, r) = \frac{1}{1 + q\|x - x^*\|} \log\left(1 + q\left|f(x) - f(x^*) + r\right|\right). \tag{5.1}$$

In non-smooth case, we obtain a local minimizer by using the Hybrid Hooke and Jeeves-Direct Method. In smooth case, we apply the PRP Conjugate Gradient Method to get the search direction and the Armijo line search to get the step size. The numerical results prove that the proposed approach is efficient.

Problem 5.1.

$$\begin{aligned} &\min \quad f(x) = \left|\frac{x-1}{4}\right| + \left|\sin\left(\pi\left(1 + \frac{x-1}{4}\right)\right)\right| + 7 \\ &\text{s.t.} \quad -10 \le x \le 10. \end{aligned} \tag{5.2}$$

The global minimum solution: $x^* = 1.0000$ and $f(x^*) = 7.0000$. In this experiment, we used an initial point $x_0 = 8$. The algorithm can successfully obtain the global minimizer. The time to reach the global minimizer is 21.7842 seconds. The numbers of the filled function and the original objective function being calculated in the algorithm are 953 and 1167, respectively.

Problem 5.2.

$$\begin{aligned} &\min \quad f(x) = |x-2|(1 + 10|\sin(x+2)|) + 3 \\ &\text{s.t.} \quad -10 \le x \le 10. \end{aligned} \tag{5.3}$$

The global minimum solution: $x^* = 2.0000$ and $f(x^*) = 3.0000$. In this experiment, we used an initial point $x_0 = -5$. The algorithm can successfully obtain the global minimizer. The time to reach the global minimizer is 23.9746 seconds. The numbers of the filled function and the original objective function being calculated in the algorithm are 8195 and 9479, respectively.

Problem 5.3.

$$\begin{aligned} &\min \quad f(x) = \max\left\{5x_1 + x_2, -5x_1 + x_2, x_1^2 + x_2^2 + 4x_2\right\} \\ &\text{s.t.} \quad -4 \le x_1 \le 4, -4 \le x_2 \le 4. \end{aligned} \tag{5.4}$$

The global minimum solution: $x^* = (0, -3)$ and $f(x^*) = -3$. In this experiment, we used an initial point $x_0 = (-4, 2)$. The algorithm can successfully obtain the global minimizer. The time to reach the global minimizer is 28.5745 seconds. The numbers of the filled function and the original objective function being calculated in the algorithm are 1986 and 2488, respectively.

Problem 5.4.

$$\begin{aligned} \min \quad & f(x) = -20\exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n}|x_i|}\right) - \exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)\right) + 20 \\ \text{s.t.} \quad & -20 \le x_i \le 30, \quad i = 1, 2, \ldots, n. \end{aligned} \tag{5.5}$$

For any n, the global minimum solution: $x^* = (0, 0, \ldots, 0)$ and $f(x^*) = -2.7183$. In this experiment, we considered $n = 10$ and used $x_0 = (-10, -10, \ldots, -10)$ as an initial point. The algorithm can successfully obtain the global minimizer. The time to reach the global minimizer is 93.6783 seconds. The numbers of the filled function and the original objective function being calculated in the algorithm are 7631 and 9739, respectively.

Problem 5.5.

$$\begin{aligned} \min \quad & f(x) = \max_{j=1,\ldots,m}\sum_{i=1}^{n}\frac{(ix_i-1)^2}{i+j-1} + \min_{j=1,\ldots,m}\sum_{i=1}^{n}\frac{(ix_i-1)^2}{i+j-1} \\ \text{s.t.} \quad & -10 \le x_i \le 10, \quad i = 1, \ldots, n. \end{aligned} \tag{5.6}$$

For any n, m, the global minimum solution: $x^* = (1, 0.5, \ldots, 0.1)$ and $f(x^*) = 0$. In this experiment, we considered $n = 15, m = 15$, and used $x_0 = (-7, -7, \ldots, -7)$ as an initial point. The algorithm can successfully obtain the global minimizer. The time to reach the global minimizer is 149.5783 seconds. The numbers of the filled function and the original objective function being calculated in the algorithm are 9761 and 14264, respectively.

Problem 5.6.

$$\begin{aligned} \min \quad & f(x) = \frac{\pi}{n}\left(10\sin^2\pi x_1 + g(x) + (x_n-1)^2\right) \\ \text{s.t.} \quad & -10 \le x_i \le 10, \quad i = 1, 2, \ldots, n, \end{aligned} \tag{5.7}$$

where $g(x) = \sum_{i=1}^{n-1}[(x_i-1)^2(1+10\sin^2\pi x_{i+1})]$. For any n, the global minimum solution: $x^* = (1, 1, \ldots, 1)$ and $f(x^*) = 0$. In this experiment, we considered $n = 20$, and used $x_0 = (7, 7, \ldots, 7)$ as an initial point. The algorithm can successfully obtain the global minimizer. The time to reach the global minimizer is 172.8436 seconds. The numbers of the filled function and the original objective function being calculated in the algorithm are 12674 and 16774, respectively.

6. Conclusions

In this paper, we first give a definition of a filled function for a non-smooth unconstrained minimization problem and construct a new filled function with two parameters. Then, we design an elaborate solution algorithm based on this filled function. Finally, we make a numerical test. The computational results suggest that this filled function approach is efficient. Of course, the efficiency of the proposed filled function approach relies on the non-smooth local optimization procedure. Meanwhile, from the numerical results, we can see that algorithm can move successively from one local minimum to another better one, but in most cases, we have to use more time to judge the current point being a global minimizer than to find a global minimizer. However, the global optimality conditions for continuous variables are still open problem, in general. The criterion of the global minimizer will provide solid stopping conditions for a continuous filled function method.

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