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Research Article

Permanence and Global Attractivity of a Discrete Two-Prey One-Predator Model with Infinite Delay

Zhixiang Yu and Zhong Li

College of Mathematics and Computer Science, Fuzhou University, Fuzhou, Fujian 350108, China

Correspondence should be addressed to Zhong Li, lizhong04108@163.com

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A discrete two-prey one-predator model with infinite delay is proposed. A set of sufficient conditions which guarantee the permanence of the system is obtained. By constructing a suitable Lyapunov functional, we also obtain sufficient conditions ensuring the global attractivity of the system. An example together with its numerical simulation shows the feasibility of the main results.

1. Introduction

The aim of this paper is to investigate the persistence and stability property of the following discrete two-prey one-predator model with infinite delays:

$$\begin{aligned} x_1(n+1) &= x_1(n)\exp\left[a_1(n) - b_1(n)x_1(n) - c_1(n)\sum_{s=-\infty}^{n} H_1(n-s)x_3(s)\right], \\ x_2(n+1) &= x_2(n)\exp\left[a_2(n) - b_2(n)x_2(n) - c_2(n)\sum_{s=-\infty}^{n} H_2(n-s)x_3(s)\right], \\ x_3(n+1) &= x_3(n)\exp\left[-b_3(n)x_3(n) + d_1(n)\sum_{s=-\infty}^{n} H_3(n-s)x_1(s)\right. \\ &\qquad \left. + \, d_2(n)\sum_{s=-\infty}^{n} H_4(n-s)x_2(s)\right], \end{aligned} \tag{1.1}$$

where $x_i(n)$, $i = 1,2$, are the densities of the prey species i at the nth generation; $x_3(n)$ is the density of the predator at the nth generation; $\{a_i(n)\}$, $\{c_i(n)\}$, $\{d_i(n)\}(i = 1,2)$, $\{b_i(n)\}$

$(i = 1,2,3)$; $\{H_i(n)\}(i = 1,\dots,4)$ are all bounded nonnegative sequences such that

$$0 < a_i^l \le a_i^u, \quad 0 < c_i^l \le c_i^u, \quad 0 < d_i^l \le d_i^u, \quad 0 < b_i^l \le b_i^u, \quad \sum_{n=0}^{\infty} H_i(n) = 1. \tag{1.2}$$

Here, for any bounded sequence $\{g(n)\}$, set $g^u = \sup_{n\in N}\{g(n)\}$ and $g^l = \inf_{n\in N}\{g(n)\}$.

From the point of view of biology, in the sequel, we assume that

$$x_i(s) = \Phi_i(s) \ge 0, \quad \Phi_i(0) > 0, \; i = 1,2,3, \tag{1.3}$$

where $s = \dots,-n,-n+1,\dots,-1,0$. Then system (1.1) with the initial condition (1.3) has a unique positive solution $(x_1(n), x_2(n), x_3(n))^T$.

As one of the dominant themes in mathematical biology, the predator-prey relationship has been studied in a number of ways (see [1–4] and the references therein). In 1970, Parrish and Saila [5] firstly proposed the one-prey two-predator model as follows:

$$\begin{aligned} \dot{x}_1(t) &= x_1(t)\big(b_1 - x_1(t) - \alpha x_2(t) - \eta z(t)\big), \\ \dot{x}_2(t) &= x_2(t)\big(b_2 - \beta x_1(t) - x_2(t) - \upsilon z(t)\big), \\ \dot{z}(t) &= z(t)\big(-b_3 + d\eta x_1(t) + d\upsilon x_2(t)\big). \end{aligned} \tag{1.4}$$

Gramer and May [6] studied the stability of the positive equilibrium of system (1.4); Takcuchi and Adachi [7] investigated the existence of the positive equilibrium and Hopf Bifurcation of the above system.

Recently, Elettreby [8] proposed the following two-prey one-predator model:

$$\begin{aligned} \dot{x}(t) &= ax(t)(1 - x(t)) - (x(t)z(t)), \\ \dot{y}(t) &= by(t)\big(1 - y(t)\big) - \big(y(t)z(t)\big), \\ \dot{z}(t) &= -cz(t)^2 + dx(t)z(t) + ey(t)z(t), \end{aligned} \tag{1.5}$$

where all the parameters in system (1.5) are positive constants. By applying differential inequality theory and iterative scheme, he showed that the unique positive equilibrium of system (1.5) is globally attractive. It is well known that a suitable ecosystem should incorporate some pase of the state of system, which is represented by time delays. Li et al. [9] studied the two-prey one-predator model with delays:

$$\begin{aligned} \dot{x}_1(t) &= x_1(t)\big(b_1 - x_1(t) - \alpha x_2(t) - r_1 y(t)\big), \\ \dot{x}_2(t) &= x_2(t)\big(b_2 - \beta x_1(t) - x_2(t) - r_2 y(t)\big), \\ \dot{y}(t) &= -b_3 y(t) + dr_1 x_1(t-\tau) y(t-\tau) + dr_2 x_2(t-\tau) y(t-\tau), \end{aligned} \tag{1.6}$$

where $x_1(t)$, $x_2(t)$, $y(t)$ are the densities of the prey and predator at the time t, respectively, b_1, b_2, b_3, α, β, r_1, r_2, d are all positive constants. They investigated the Hopf bifurcation of system (1.6). Corresponding to system (1.5), Huang [10] proposed and studied the following system with infinite delays:

$$\begin{aligned}
\dot{x}_1(t) &= ax_1(t)\left(1 - x_1(t) - \frac{1}{a}\int_{-\infty}^{t} K_1(t-s)x_3(s)ds\right),\\
\dot{x}_2(t) &= bx_2(t)\left(1 - x_2(t) - \frac{1}{b}\int_{-\infty}^{t} K_2(t-s)x_3(s)ds\right),\\
\dot{x}_3(t) &= x_3(t)\left(-cx_3(t) + d\int_{-\infty}^{t} K_3(t-s)x_1(s)ds + e\int_{-\infty}^{t} K_4(t-s)x_2(s)ds\right),
\end{aligned} \tag{1.7}$$

where all the coefficients a, b, c, d, e are positive constants, and $K_i : [0, +\infty) \to (0, +\infty)$ are continue functions such that $\int_0^{\infty} K_i(s)ds = 1$, $i = 1, 2, 3, 4$. By applying iterative scheme, he showed that the unique positive equilibrium of the system is globally attractive.

On the other hand, it is well known that the discrete time model governed by difference equations are more appropriate than the continuous ones when the populations have nonoverlapping generations. Corresponding to traditional continuous Logistic model governed by differential equations, Mohamad and Gopalsamy [11] proposed the following single species discrete model:

$$x(n+1) = x(n)\exp\left[r(n)\left(1 - \frac{x(n)}{K(n)}\right)\right]. \tag{1.8}$$

They tried to obtain a set of sufficient conditions which ensure that (1.8) admits a unique positive and globally asymptotically stable almost periodic solution. However, Zhou and Zou [12] gave an counterexample which shows that the main results of [11] are not correct. By developing some new analysis technique, Zhou and Zou [12] obtained sufficient conditions which ensure the existence of a positive and globally asymptotically stable ω-periodic solution of system (1.8). Chen and Zhou [13] further generalized system (1.8) to the following two-species Lotka-Volterra competition system:

$$\begin{aligned}
x(n+1) &= x(n)\exp\left[r_1(n)\left(1 - \frac{x(n)}{K_1(n)} - \mu_2(n)y(n)\right)\right],\\
y(n+1) &= y(n)\exp\left[r_2(n)\left(1 - \mu_1(n)x(n) - \frac{y(n)}{K_2(n)}\right)\right].
\end{aligned} \tag{1.9}$$

They obtained the sufficient conditions which guarantee the persistence of the system (1.3). Also, for the periodic case, they obtained the sufficient conditions which guarantee the existence of a globally stable periodic solution of the system. Wang and Lu [14] proposed the following Lotka-Volterra model:

$$x_i(k+1) = x_i(k)\exp\left[r_i(k) - \sum_{j=1}^{n} a_{ij}(k)x_j(k)\right], \quad i = 1, 2, \ldots, n, \tag{1.10}$$

where $x_i(k)$ is the density of population i at kth generation; $r_i(k)$ is the growth rate of population i at kth generation; $a_{ij}(k)$ measures the intensity of intraspecific competition or interspecific action of species. By constructing a suitable Lyapunov function and using the finite covering theorem of Mathematic Analysis, they obtained a set of sufficient conditions which ensures the system to be globally asymptotically stable. Similar to the continuous ones, some scholars also argued that the discrete model should incorporate some past state of the species and thus should consider the discrete model with time delay. Recently, Chen [15] investigated the persistent property of the following discrete two species Lotka-Volterra competition model with deviating arguments:

$$\begin{aligned} x_1(n+1) &= x_1(n)\exp\left[r_1(n)\left(1-\frac{x_1(n)}{K_1(n)}-\mu_2(n)\sum_{s=-\infty}^{n}H_1(n-s)x_2(s)\right)\right],\\ x_2(n+1) &= x_2(n)\exp\left[r_2(n)\left(1-\mu_1(n)\sum_{s=-\infty}^{n}H_2(n-s)x_1(s)-\frac{x_2(n)}{K_2(n)}\right)\right], \end{aligned} \tag{1.11}$$

where $x_i(n)$, $i = 1,2$, are the densities of competition species i at nth generation. By establishing a new difference inequality, Chen [15] showed that under the same conditions as that of Chen and Zhou [13], (1.11) is also permanent, which means that with some suitable restriction on the coefficients of the system, delay has no influence on the persistent property of the system. Chen [16] also investigated the persistent property of a discrete n-species nonautonomous Lotka-Volterra competitive systems with infinite delays and feedback controls. As we can see, both [15] and [16] considered the persistent property of the system, but they did not investigate the stability property of the system. Recently, Chen et al. [17] investigated the dynamic behaviors of the following general discrete nonautonomous system of plankton allelopathy with finite time delay:

$$\begin{aligned} N_1(k+1) &= N_1(k)\exp\left[r_1(k)-\sum_{l=0}^{m}a_{1l}(k)N_1(k-l)-\sum_{l=0}^{m}b_{1l}(k)N_2(k-l)\right.\\ &\qquad\left.-\sum_{l=0}^{m}c_{1l}(k)N_1(k)N_2(k-l)\right],\\ N_2(k+1) &= N_2(k)\exp\left[r_2(k)-\sum_{l=0}^{m}a_{2l}(k)N_2(k-l)-\sum_{l=0}^{m}b_{2l}(k)N_1(k-l)\right.\\ &\qquad\left.-\sum_{l=0}^{m}c_{2l}(k)N_2(k)N_1(k-l)\right], \end{aligned} \tag{1.12}$$

where $N_i(k)$ represents the densities of population i at the kth generation; $r_i(k)$ is the intrinsic growth rate of population i at the kth generation; $a_{il}(k)$ measures the intraspecific influence of the $(k-l)$th generation of population i on the density of own population; $b_{il}(k)$ stands for the inter-specific influence of the $(k-l)$th generation of population i on the density of own population and $c_{il}(k)$ stands for the effect of toxic inhibition of population i by population j at the $(k-l)$th generation, $i,j = 1,2$ and $i \neq j$. $\{r_i(k)\}$, $\{a_{il}(k)\}$, $\{b_{il}(k)\}$ and $\{c_{il}(k)\}$ are all bounded nonnegative sequences defined for $k \in N$. They obtained sufficient conditions which guarantee the permanence, global attractivity and partial extinction of the above system.

Concerned with the discrete predator-prey system, by giving the detail analysis of the right hand side of the system, Yang [18] obtained a set of sufficient conditions which ensures the uniform persistence of the system investigated. Recently, Chen and Chen [19] proposed the following discrete periodic Volterra model with mutual interference and Holling II type functional response

$$\begin{aligned} x(n+1) &= x(n)\exp\left[r_1(n) - b_1(n)x(n) - \frac{c_1(n)}{k+x(n)}y^m(n)\right], \\ y(n+1) &= y(n)\exp\left[-r_2(n) - b_2(n)y(n) - \frac{c_2(n)x(n)}{k+x(n)}y^{m-1}(n)\right]. \end{aligned} \tag{1.13}$$

They also obtained sufficient conditions which ensure the permanence of the system. For more works on discrete population model, one could refer to [11–42] and the references cited therein.

However, to the best of the authors knowledge, to this day, no scholars propose and study the discrete predator-prey model with infinite delays. This motivates us to propose and study (1.1). The aim of this paper is to investigate the persistent and stability property of system (1.1).

The rest of the paper is arranged as follows: some useful lemmas are stated in the following section. Sufficient conditions which ensure the permanence and global attractivity of system (1.1) are stated and proved in Section 3. In Section 4, an example together with its numeric simulation shows the feasibility of the main results. We end this paper by a brief discussion.

2. Preliminaries

Now let us state several lemmas which will be useful in proving main results.

Lemma 2.1 (see [29]). *Assume that* $\{x(n)\}$ *satisfies* $x(n) > 0$ *and*

$$x(n+1) \le x(n)\exp[r(n) - a(n)x(n)], \tag{2.1}$$

for $n \in N$, *where* $\{r(n)\}$ *and* $\{a(n)\}$ *are all positive sequences bounded above and below by positive constants. Then*

$$\limsup_{n\to+\infty} x(n) \le \frac{1}{a^l}\exp[r^u - 1]. \tag{2.2}$$

Lemma 2.2 (see [29]). *Assume that* $\{x(n)\}$ *satisfies*

$$x(n+1) \ge x(n)\exp[r(n) - a(n)x(n)], \quad k \ge N_0, \tag{2.3}$$

$\limsup_{n\to+\infty} x(n) \le x^*$ *and* $x(N_0) > 0$, *where* $\{r(n)\}$ *and* $\{a(n)\}$ *are all positive sequences bounded above and below by positive constants and* $N_0 \in N$. *Then*

$$\liminf_{n\to+\infty} x(n) \ge \frac{r^l \exp\left[r^l - a^u x^*\right]}{a^u}. \tag{2.4}$$

Lemma 2.3 (see [43]). *Let* $x : Z \to R$ *be a nonnegative bounded sequences, and let* $H : N \to R$ *be a nonnegative sequences such that* $\sum_{n=0}^{\infty} H(n) = 1$. *Then*

$$\liminf_{n\to+\infty} x(n) \le \liminf_{n\to+\infty} \sum_{s=-\infty}^{n} H(n-s)x(s) \le \limsup_{n\to+\infty} \sum_{s=-\infty}^{n} H(n-s)x(s) \le \limsup_{n\to+\infty} x(n). \tag{2.5}$$

3. Main Results

Now, we investigate the persistence property and stability property of system (1.1).

Theorem 3.1. *Assume that*

$$a_1^l - c_1^u M_3 > 0, \tag{H_1}$$

$$a_2^l - c_2^u M_3 > 0 \tag{H_2}$$

hold, then system (1.1) *is permanent, that is,*

$$m_i \le \liminf_{n\to+\infty} x_i(n) \le \limsup_{n\to+\infty} x_i(n) \le M_i, \quad i = 1,2,3, \tag{3.1}$$

where

$$\begin{gathered} M_i = \frac{1}{b_i^l} \exp\left[a_i^u - 1\right], \quad i = 1,2, \qquad M_3 = \frac{1}{b_3^l} \exp\left[d_1^u M_1 + d_2^u M_2 - 1\right], \\ m_i = \frac{a_i^l - c_i^u M_3}{b_i^u} \exp\left[a_i^l - c_i^u M_3 - b_i^u M_i\right], \quad i = 1,2, \\ m_3 = \frac{d_1^l m_1 + d_2^l m_2}{b_3} \exp\left[d_1^l m_1 + d_2^l m_2 - b_3^u M_3\right]. \end{gathered} \tag{3.2}$$

Proof. It follows from the first two equations of system (1.1) that

$$x_i(n+1) \le x_i(n) \exp\left[a_i(n) - b_i(n)x_i(n)\right], \quad i = 1,2. \tag{3.3}$$

So, as a consequence of Lemma 2.1, for any solution $(x_1(n), x_2(n), x_3(n))_{n=0}^{\infty}$ of system (1.1) with initial condition (1.3), one has

$$\limsup_{n\to+\infty} x_i(n) \le \frac{1}{b_i^l} \exp\left[a_i^u - 1\right] \overset{\text{def}}{=} M_i, \quad i = 1,2. \tag{3.4}$$

According to Lemma 2.3, from (3.4), we have

$$\limsup_{n\to+\infty}\sum_{s=-\infty}^{n} H_3(n-s)x_1(s) \le M_1, \qquad \limsup_{n\to+\infty}\sum_{s=-\infty}^{n} H_4(n-s)x_2(s) \le M_2. \tag{3.5}$$

For any small positive constant $\varepsilon > 0$, it follows from (3.5) that there exists a positive integer n_1 such that for all $n \ge n_1$,

$$\sum_{s=-\infty}^{n} H_3(n-s)x_1(s) \le M_1+\varepsilon, \qquad \sum_{s=-\infty}^{n} H_4(n-s)x_2(s) \le M_2+\varepsilon. \tag{3.6}$$

Thus, for all $n \ge n_1$, from the last equation of system (1.1), if follows that

$$x_3(n+1) \le x_3(n)\exp\Big[-b_3^l x_3(n) + d_1^u(M_1+\varepsilon) + d_2^u(M_2+\varepsilon)\Big]. \tag{3.7}$$

By applying Lemma 2.1 to (3.7), we have

$$\limsup_{n\to+\infty} x_3(n) \le \frac{1}{b_3^l}\exp\big[d_1^u(M_1+\varepsilon) + d_2^u(M_2+\varepsilon) - 1\big]. \tag{3.8}$$

Setting $\varepsilon \to 0$, it follows that

$$\limsup_{n\to+\infty} x_3(n) \le \frac{1}{b_3^l}\exp\big[d_1^u M_1 + d_2^u M_2 - 1\big] \stackrel{\text{def}}{=} M_3. \tag{3.9}$$

Next, we show that under the assumption of Theorem 3.1,

$$\liminf_{n\to+\infty} x_i(n) \ge m_i, \quad i=1,2,3. \tag{3.10}$$

According to Lemma 2.3, from (3.9) we have

$$\limsup_{n\to\infty}\sum_{s=-\infty}^{n} H_i(n-s)x_3(s) \le M_3, \quad i=1,2. \tag{3.11}$$

Conditions (H_1) and (H_2) imply that for enough small positive constant ε_1, we have

$$a_i^l - C_i^u(M_3+\varepsilon_1) > 0, \quad i=1,2. \tag{3.12}$$

For ε_1, it follows from (3.11) that there exists an positive integer $n_2 \ge n_1$ such that for all $n > n_2$,

$$\sum_{s=-\infty}^{n} H_i(n-s)x_3(s) \le M_3+\varepsilon_1, \quad i=1,2. \tag{3.13}$$

For $n > n_2$, from (3.13) and the first two equations of system (1.1), we have

$$x_i(n+1) \geq x_i(n) \exp\left[a_i^l - b_i^u x_i(n) - c_i^u (M_3 + \varepsilon_1)\right], \quad i = 1, 2. \tag{3.14}$$

Thus, according to Lemma 2.2, one has

$$\liminf_{n \to +\infty} x_i(n) \geq \frac{a_i^l - c_i^u (M_3 + \varepsilon_1)}{b_i^u} \exp\left[a_i^l - c_i^u (M_3 + \varepsilon_1) - b_i^u M_i\right], \quad i = 1, 2. \tag{3.15}$$

Setting $\varepsilon_1 \to 0$, it follows that

$$\liminf_{n \to +\infty} x_i(n) \geq \frac{a_i^l - c_i^u M_3}{b_i^u} \exp\left[a_i^l - c_i^u M_3 - b_i^u M_i\right] \stackrel{\text{def}}{=} m_i, \quad i = 1, 2. \tag{3.16}$$

According to Lemma 2.3, from (3.16) we have

$$\liminf_{n \to +\infty} \sum_{s=-\infty}^{n} H_3(n-s) x_1(s) \geq m_1, \qquad \liminf_{n \to +\infty} \sum_{s=-\infty}^{n} H_4(n-s) x_2(s) \geq m_2. \tag{3.17}$$

For any $\varepsilon_2 > 0$ small enough, without loss of generality, we may assume that $\varepsilon_2 < (1/2)\min\{m_1, m_2\}$. From (3.17), it follows that there exists a n_3, such that for all $n \geq n_3$,

$$\sum_{s=-\infty}^{n} H_3(n-s) x_1(s) \geq m_1 - \varepsilon_2, \qquad \sum_{s=-\infty}^{n} H_4(n-s) x_2(s) \geq m_2 - \varepsilon_2. \tag{3.18}$$

For $n \geq n_3$, from (3.18) and the last equation of (1.1), we have

$$x_3(n+1) \geq x_3(n) \exp\left[-b_3^u x_3(n) + d_1^l (m_1 - \varepsilon_2) + d_2^l (m_2 - \varepsilon_2)\right]. \tag{3.19}$$

By applying Lemma 2.2 to (3.19), it follows that

$$\begin{aligned} \liminf_{n \to +\infty} x_3(n) \geq{}& \frac{d_1^l (m_1 - \varepsilon_2) + d_2^l (m_2 - \varepsilon_2)}{b_3^u} \\ & \times \exp\left[d_1^l (m_1 - \varepsilon_2) + d_2^l (m_2 - \varepsilon_2) - b_3^u M_3\right]. \end{aligned} \tag{3.20}$$

Setting $\varepsilon_2 \to 0$, it follows that

$$\liminf_{n \to +\infty} x_3(n) \geq \frac{d_1^l m_1 + d_2^l m_2}{b_3^u} \exp\left[d_1^l m_1 + d_2^l m_2 - b_3^u M_3\right] \stackrel{\text{def}}{=} m_3. \tag{3.21}$$

This ends the proof of Theorem 3.1. □

Theorem 3.2. *Assume that* (H_1) *and* (H_2) *hold. Assume further that there exist positive constants* α, β, γ *and* δ *such that*

$$\alpha \min\left\{b_1^l, \frac{2}{M_1} - b_1^u\right\} - \gamma d_1^u > \delta, \tag{H_3}$$

$$\beta \min\left\{b_2^l, \frac{2}{M_2} - b_2^u\right\} - \gamma d_2^u > \delta, \tag{H_4}$$

$$\gamma \min\left\{b_3^l, \frac{2}{M_3} - b_3^u\right\} - \alpha c_1^u - \beta c_2^u > \delta, \tag{H_5}$$

hold. Then for any two positive solutions $(x_1(n), x_2(n), x_3(n))^T$ *and* $(y_1(n), y_2(n), y_3(n))^T$ *of system* (1.1)*, one has*

$$\lim_{n \to +\infty} \left(x_i(n) - y_i(n)\right) = 0, \quad i = 1, 2, 3. \tag{3.22}$$

Proof. From conditions (H_3)–(H_5), there exits an enough small positive constant ε_3 such that

$$\alpha \min\left\{b_1^l, \frac{2}{M_1 + \varepsilon_3} - b_1^u\right\} - \gamma d_1^u > \delta, \tag{3.23}$$

$$\beta \min\left\{b_2^l, \frac{2}{M_2 + \varepsilon_3} - b_2^u\right\} - \gamma d_2^u > \delta, \tag{3.24}$$

$$\gamma \min\left\{b_3^l, \frac{2}{M_3 + \varepsilon_3} - b_3^u\right\} - \alpha c_1^u - \beta c_2^u > \delta. \tag{3.25}$$

Since (H_1) and (H_2) hold, for any solutions $(x_1(n), x_2(n), x_3(n))^T$ and $(y_1(n), y_2(n), y_3(n))^T$ of system (1.1) with the initial conditions (1.3), it follows from Theorem 3.1 that

$$\limsup_{n \to +\infty} x_i(n) \le M_i, \quad \limsup_{n \to +\infty} y_i(n) \le M_i, \quad i = 1, 2, 3. \tag{3.26}$$

For the above ε_3 and (3.26), there exists an $n^* > 0$ such that for all $n > n^*$,

$$x_i(n) \le M_i + \varepsilon_3, \qquad y_i(n) \le M_i + \varepsilon_3. \tag{3.27}$$

Firstly, let

$$V_{11}(n) = \left|\ln x_1(n) - \ln y_1(n)\right|. \tag{3.28}$$

Then from the first equation of the system (1.1), we have

$$\begin{aligned} V_{11}(n+1) &= \left|\ln x_1(n+1) - \ln y_1(n+1)\right| \\ &\le \left|\ln x_1(n) - \ln y_1(n) - b_1(n)\left(x_1(n) - y_1(n)\right)\right| \\ &\quad + c_1(n)\left|\sum_{s=-\infty}^{n} H_1(n-s) \mid x_3(s) - y_3(s)\right|. \end{aligned} \tag{3.29}$$

Using the Mean Value Theorem, we get

$$x_1(n) - y_1(n) = \exp(\ln x_1(n)) - \exp\left(\ln y_1(n)\right) = \xi_1(n)\left(\ln x_1(n) - \ln y_1(n)\right), \tag{3.30}$$

where $\xi_1(n)$ lies between $x_1(n)$ and $y_1(n)$, then it follows that

$$\begin{aligned} V_{11}(n+1) \le \left|\ln x_1(n) - \ln y_1(n)\right| &- \left(\frac{1}{\xi_1(n)} - \left|\frac{1}{\xi_1(n)} - b_1(n)\right|\right)\left|x_1(n) - y_1(n)\right| \\ + c_1(n)\left|\sum_{s=-\infty}^{n} H_1(n-s) \mid x_3(s) - y_3(s)\right|&, \end{aligned} \tag{3.31}$$

and so

$$\begin{aligned} \Delta V_{11} \le &-\left(\frac{1}{\xi_1(n)} - \left|\frac{1}{\xi_1(n)} - b_1(n)\right|\right)\left|x_1(n) - y_1(n)\right| \\ &+ c_1(n)\left|\sum_{s=-\infty}^{n} H_1(n-s) \mid x_3(s) - y_3(s)\right|. \end{aligned} \tag{3.32}$$

Secondly, let

$$V_{12}(n) = \sum_{s=0}^{\infty} H_1(s) \sum_{\theta=n-s}^{n-1} c_1(\theta+s)\left|x_3(\theta) - y_3(\theta)\right|. \tag{3.33}$$

then, similar to the aforementioned analysis, we have

$$\begin{aligned} \Delta V_{12} &= V_{12}(n+1) - V_{12}(n) \\ &\le c_1^u\left|x_3(n) - y_3(n)\right| - c_1(n) \sum_{s=-\infty}^{n} H_1(n-s)\left|x_3(s) - y_3(s)\right|. \end{aligned} \tag{3.34}$$

Now, set

$$V_1(n) = V_{11}(n) + V_{12}(n), \tag{3.35}$$

then from (3.32) and (3.34), we have

$$\begin{aligned}\Delta V_1 &= \Delta V_{11} + \Delta V_{12} \\ &\leq -\left(\frac{1}{\xi_1(n)} - \left|\frac{1}{\xi_1(n)} - b_1(n)\right|\right)\left|x_1(n) - y_1(n)\right| \\ &\quad + c_1^u\left|x_3(n) - y_3(n)\right|.\end{aligned} \tag{3.36}$$

Let

$$V_2(n) = V_{21}(n) + V_{22}(n), \tag{3.37}$$

where

$$\begin{aligned}V_{21}(n) &= \left|\ln x_2(n) - \ln y_2(n)\right|, \\ V_{22}(n) &= \sum_{s=0}^{\infty} H_2(s) \sum_{\theta=n-s}^{n-1} c_2(\theta+s)\left|x_3(\theta) - y_3(\theta)\right|.\end{aligned} \tag{3.38}$$

Similar to the aforementioned analysis, we have

$$\begin{aligned}\Delta V_2 &= \Delta V_{21} + \Delta V_{22} \\ &\leq -\left(\frac{1}{\xi_2(n)} - \left|\frac{1}{\xi_2(n)} - b_2(n)\right|\right)\left|x_2(n) - y_2(n)\right| + c_2^u\left|x_3(n) - y_3(n)\right|,\end{aligned} \tag{3.39}$$

where $\xi_2(n)$ lies between $x_2(n)$ and $y_2(n)$.

Let

$$V_3(n) = V_{31}(n) + V_{32}(n), \tag{3.40}$$

where

$$\begin{aligned}V_{31}(n) &= \left|\ln x_3(n) - \ln y_3(n)\right|, \\ V_{32}(n) &= \sum_{s=0}^{\infty} H_3(s) \sum_{\theta=n-s}^{n-1} d_1(\theta+s)\left|x_1(\theta) - y_1(\theta)\right| \\ &\quad + \sum_{s=0}^{\infty} H_4(s) \sum_{\theta=n-s}^{n-1} d_2(\theta+s)\left|x_2(\theta) - y_2(\theta)\right|.\end{aligned} \tag{3.41}$$

Similar to the aforementioned analysis, we have

$$\begin{aligned}\Delta V_3 &= \Delta V_{31} + \Delta V_{32}\\ &\leq -\left(\frac{1}{\xi_3(n)} - \left|\frac{1}{\xi_3(n)} - b_3(n)\right|\right)\left|x_3(n) - y_3(n)\right| \\ &\quad + d_1^u\left|x_1(n) - y_1(n)\right| + d_2^u\left|x_2(n) - y_2(n)\right|,\end{aligned} \tag{3.42}$$

where $\xi_3(n)$ lies between $x_3(n)$ and $y_3(n)$.

Now, we define a Lyapunou functional as follows:

$$V(n) = \alpha V_1(n) + \beta V_2(n) + \gamma V_3(n). \tag{3.43}$$

Calculating the difference of V along the solution of system (1.1), for $n > n^*$, it follows from (3.23), (3.24), (3.25), (3.27), (3.36), (3.39) and (3.42) that

$$\begin{aligned}\Delta V &\leq -\left[\alpha \min\left\{b_1^l, \frac{2}{M_1+\varepsilon_3} - b_1^u\right\} - \gamma d_1^u\right]\left|x_1(n) - y_1(n)\right| \\ &\quad - \left[\beta \min\left\{b_2^l, \frac{2}{M_2+\varepsilon_3} - b_2^u\right\} - \gamma d_2^u\right]\left|x_2(n) - y_2(n)\right| \\ &\quad - \left[\gamma \min\left\{b_3^l, \frac{2}{M_3+\varepsilon_3} - b_3^u\right\} - \alpha c_1^u - \beta c_2^u\right]\left|x_3(n) - y_3(n)\right| \\ &\leq -\delta\sum_{i=1}^{n}\left|x_i(n) - y_i(n)\right|.\end{aligned} \tag{3.44}$$

Summating both sides of the above inequalities from n^* to n, we have

$$\sum_{p=n^*}^{n}\left(V(p+1) - V(p)\right) \leq -\delta\sum_{p=n^*}^{n}\sum_{i=1}^{3}\left|x_i(p) - y_i(p)\right|, \tag{3.45}$$

which implies

$$V(n+1) + \delta\sum_{p=n^*}^{n}\sum_{i=1}^{3}\left|x_i(p) - y_i(p)\right| \leq V(n^*). \tag{3.46}$$

It follows that

$$\sum_{p=n^*}^{n}\sum_{i=1}^{3}\left|x_i(p)-y_i(p)\right| \le \frac{V(n^*)}{\delta}, \tag{3.47}$$

then

$$\sum_{p=n^*}^{n}\sum_{i=1}^{3}\left|x_i(p)-y_i(p)\right| \le \frac{V(n^*)}{\delta} < +\infty, \tag{3.48}$$

which means that $\lim_{n\to\infty}\sum_{i=1}^{3}|x_i(n)-y_i(n)| = 0$, that is

$$\lim_{n\to+\infty}\left(x_i(n)-y_i(n)\right) = 0, \quad i = 1,2,3. \tag{3.49}$$

This completes the proof of Theorem 3.2. □

4. Example

The following example shows the feasibility of the main results.

Example 4.1. Consider the following system:

$$\begin{aligned}
x_1(n+1) &= x_1(n)\exp\Bigg[\left(1.1+0.1\sin\left(\sqrt{2}n\right)\right)-\left(1.2+0.2\sin\left(\sqrt{2}n\right)\right)x_1(n) \\
&\quad -(0.3+0.2\cos(n))\sum_{s=-\infty}^{n}\frac{e-1}{e}e^{-(n-s)}x_3(s)\Bigg], \\
x_2(n+1) &= x_2(n)\exp\Bigg[\left(1+0.1\sin\left(\sqrt{3}n\right)\right)-(1.1+0.1\sin(n))x_2(n) \\
&\quad -\left(0.2+0.02\cos\left(\sqrt{3}n\right)\right)\sum_{s=-\infty}^{n}\frac{e^2-1}{e^2}e^{-2(n-s)}x_3(s)\Bigg], \\
x_3(n+1) &= x_3(n)\exp\Bigg[-\left(1.15+0.15\sin\left(\sqrt{3}n\right)\right)x_3(n) \\
&\quad +\left(0.12+0.03\cos\left(\sqrt{2}n\right)\right)\sum_{s=-\infty}^{n}\frac{e^3-1}{e^3}e^{-3(n-s)}x_1(s) \\
&\quad +(0.13+0.03\cos(n))\sum_{s=-\infty}^{n}\frac{e^4-1}{e^4}e^{-4(n-s)}x_2(s)\Bigg].
\end{aligned} \tag{4.1}$$

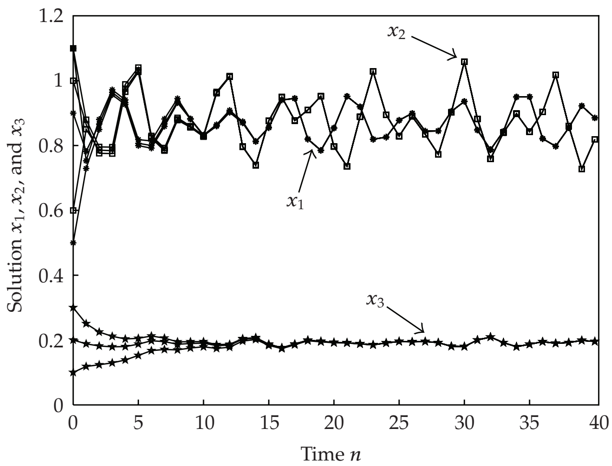

Figure 1: Dynamics behaviors of system (4.1) with initial conditions $(x_1(s), x_2(s), x_3(s))^T = (0.5, 1, 0.3)^T, (1.1, 1.1, 0.1)^T, (0.9, 0.6, 0.2)^T$, $s = \ldots, -n, -n+1, \ldots, -1, 0$.

One could easily see that there exist $\alpha = 0.06$, $\beta = 0.05$, $\gamma = 0.05$ and $\delta = 0.001$ such that

$$\begin{aligned} a_1^l - c_1^u M_3 &= 1 - 0.5M - 3 \approx 0.7364 > 0, \\ a_2^l - c_2^u M_3 &= 1 - 0.22M_3 \approx 0.8840 > 0, \\ \alpha \min\left\{ b_1^l, \frac{2}{M_1} - b_1^u \right\} - \gamma d_1^u &\approx 0.0082 > \delta, \\ \beta \min\left\{ b_2^l, \frac{2}{M_2} - b_2^u \right\} - \gamma d_2^u &\approx 0.0241 > \delta, \\ \gamma \min\left\{ b_3^l, \frac{2}{M_3} - b_3^u \right\} - \alpha c_1^u - \beta c_2^u &\approx 0.009 > \delta. \end{aligned} \tag{4.2}$$

Clearly, conditions (H_1)–(H_5) are satisfied. From Theorems 3.1 and 3.2, (1.1) is permanent and globally attractive. Numeric simulation (Figure 1) strongly supports our results.

5. Discussion

In this paper, we propose a discrete two-prey one-predator model with infinite delay. Theorem 3.1 shows that to ensure the permanence of the system, one should ensure a_i^l, $i = 1, 2$ and b_3^l enough large, that is, the net birth rate of prey species and the density restriction of predator species should be large enough. We also obtain a set of sufficient conditions which ensures the global attractivity of the system.

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