Hindawi Publishing Corporation
Discrete Dynamics in Nature and Society
Volume 2009, Article ID 243974, 12 pages
doi:10.1155/2009/243974

Research Article

Double Periodic Solutions for a Ratio-Dependent Predator-Prey System with Harvesting Terms on Time Scales

Zhenjie Liu

School of Mathematics and Computer, Harbin University, Harbin, Heilongjiang 150086, China

Correspondence should be addressed to Zhenjie Liu, liouj2008@126.com

Received 6 April 2009; Accepted 1 September 2009

Recommended by Guang Zhang

We investigate a nonautonomous ratio-dependent predator-prey model with Beddington-DeAngelis functional response and multiple harvesting (or exploited) terms on time scales. By means of using a continuation theorem based on coincidence degree theory, we obtain sufficient criteria for the existence of at least two periodic solutions for the system. Moreover, when the time scale $\mathbb{T}$ is chosen as $\mathbb{R}$ or $\mathbb{Z}$, the existence of the periodic solutions of the corresponding continuous and discrete models follows. Therefore, the methods are unified to provide the existence of the desired solutions for the continuous differential equations and discrete difference equations.

1. Introduction

In recent years, the existence of periodic solutions for the predator-prey model has been widely studied. Models with harvesting (or exploited) terms are often considered (see, e.g., [1–4]). Generally, the model with harvesting (or exploited) terms is described as follows:

$$\begin{aligned} \dot{x} &= xf(x,y) - h, \\ \dot{y} &= yg(x,y) - k, \end{aligned} \tag{1.1}$$

where x and y are functions of time representing densities of prey and predator, respectively; h and k are harvesting (or exploited) terms standing for the harvests (see [5]). Considering the inclusion of the effect of changing environment, Zhang and Hou [6] considered the following model of ordinary differential equations with Holling-type II functional response and harvesting (or exploited) terms:

$$\dot{x}(t) = x(t)\left(a(t) - b(t)x(t) - \frac{c(t)y(t)}{m(t)y(t) + x(t)}\right) - h_1(t),$$
$$\dot{y}(t) = y(t)\left(-d(t) + \frac{f(t)x(t)}{m(t)y(t) + x(t)}\right) - h_2(t), \tag{1.2}$$

where the parameters in system (1.2) are continuous positive ω-periodic functions. Authors discussed the existence of positive periodic solutions of system (1.2) in the region $D = \{(x, y) \mid x > 0,\ y > 0\}$.

On the other hand, the theory of calculus on time scales unifies continuous and discrete analysis, many results concerning differential equations carry over quite easily to corresponding results for difference equations, while other results seem to be completely different from their continuous counterparts. The study of dynamic equations on general time scales can reveal such discrepancies and help avoid proving results twice-once for differential equations and once again for difference equations. The two main features of the calculus on time scales are unification and extension. To prove a result for a dynamic equation on a time scale is not only related to the set of real numbers or set of integers but those pertaining to more general time scales.

The principle aim of this paper is to systematically unify the existence of multiple periodic solutions of population models modelled by ordinary differential equations and their discrete analogues in form of difference equations and to extend these results to more general time scales. The approach is based on a continuation theorem in coincidence degree, which has been widely applied to deal with the existence of periodic solutions of differential equations and difference equations. Therefore, we consider the following ratio-dependent predator-prey system with Beddington-DeAngelis functional response and harvesting terms on time scales $\mathbb{T}$:

$$z_1^{\Delta}(t) = a(t) - b(t)e^{z_1(t)} - \frac{c_1(t)e^{z_2(t)}}{e^{z_2(t)} + \alpha(t)e^{z_1(t)} + \beta(t)e^{2z_2(t)}} - h_1(t)e^{-z_1(t)},$$
$$z_2^{\Delta}(t) = -d(t) + \frac{c_2(t)e^{z_1(t)}}{e^{z_2(t)} + \alpha(t)e^{z_1(t)} + \beta(t)e^{2z_2(t)}} - h_2(t)e^{-z_2(t)}, \tag{1.3}$$

where $z_1(t)$ and $z_2(t)$ represent the prey and the predator population, respectively; $a(t)$, $b(t)$, $c_i(t)$ $(i = 1, 2)$, $d(t)$, $h_i(t)$ $(i = 1, 2)$, and $\alpha(t)$, $\beta(t)$ are all rd-continuous positive ω-periodic functions denoting the prey intrinsic growth rate, death rate, capture rate, conversion rate of predator, death rate of predator, harvesting rate, and Beddington-DeAngelis functional response parameters, respectively.

Remark 1.1. In (1.3), set $x_i(t) = \exp\{z_i(t)\}$, $i = 1, 2$. If $\mathbb{T} = \mathbb{R}$, then (1.3) reduces to the ratio-dependent predator-prey system with Beddington-DeAngelis functional response and harvesting terms governed by the ordinary differential equations

$$\begin{aligned} x_1'(t) &= x_1(t)\left(a(t) - b(t)x_1(t) - \frac{c_1(t)x_2(t)}{x_2(t) + \alpha(t)x_1(t) + \beta(t)x_2^2(t)}\right) - h_1(t), \\ x_2'(t) &= x_2(t)\left(-d(t) + \frac{c_2(t)x_1(t)}{x_2(t) + \alpha(t)x_1(t) + \beta(t)x_2^2(t)}\right) - h_2(t). \end{aligned} \tag{1.4}$$

If $\mathbb{T} = \mathbb{Z}$, then (1.3) is reformulated as

$$\begin{aligned} x_1(k+1) &= x_1(k)\left(a(k) - b(k)x_1(k) - \frac{c_1(k)x_2(k)}{x_2(k) + \alpha(k)x_1(k) + \beta(k)x_2^2(k)}\right) - h_1(k), \\ x_2(k+1) &= x_2(k)\left(-d(k) + \frac{c_2(k)x_1(k)}{x_2(k) + \alpha(k)x_1(k) + \beta(k)x_2^2(k)}\right) - h_2(k), \end{aligned} \tag{1.5}$$

which is the discrete time ratio-dependent predator-prey system with Beddington-DeAngelis functional response and harvesting terms and is also a discrete analogue of (1.3).

2. Preliminaries

A time scale $\mathbb{T}$ is an arbitrary nonempty closed subset of the real numbers $\mathbb{R}$. Throughout this paper, we assume that the time scale $\mathbb{T}$ is unbounded above and below, such as $\mathbb{R}$, $\mathbb{Z}$, and $\bigcup_{k\in\mathbb{Z}}[2k, 2k+1]$. The following definitions and lemmas can be found in [7].

Definition 2.1 (see [8]). One says that a time scale $\mathbb{T}$ is periodic if there exists $p > 0$ such that if $t \in \mathbb{T}$, then $t \pm p \in \mathbb{T}$. For $\mathbb{T} \neq \mathbb{R}$, the smallest positive p is called the period of the time scale.

Definition 2.2 (see [8]). Let $\mathbb{T} \neq \mathbb{R}$ be a periodic time scale with period p. One says that the function $f : \mathbb{T} \to \mathbb{R}$ is periodic with period ω if there exists a natural number n such that $\omega = np$, $f(t+\omega) = f(t)$ for all $t \in \mathbb{T}$, and ω is the smallest number such that $f(t+\omega) = f(t)$. If $\mathbb{T} = \mathbb{R}$, one says that f is periodic with period $\omega > 0$ if ω is the smallest positive number such that $f(t+\omega) = f(t)$ for all $t \in \mathbb{T}$.

Notation 2.3. To facilitate the discussion below, we now introduce some notation to be used throughout this paper. Let $\mathbb{T}$ be ω-periodic; that is, $t \in \mathbb{T}$ implies $t + \omega \in \mathbb{T}$,

$$\begin{gathered} \kappa = \min\{[0, +\infty) \cap \mathbb{T}\}, \qquad I_\omega = [\kappa, \kappa + \omega] \cap \mathbb{T}, \\ f^M = \sup_{t\in\mathbb{T}} f(t), \qquad f^L = \inf_{t\in\mathbb{T}} f(t), \end{gathered} \tag{2.1}$$

where $f \in C_{\mathrm{rd}}(\mathbb{T}, \mathbb{R})$ is an ω-periodic function.

Notation 2.4. Let X, Z be two Banach spaces, let $L : \mathrm{Dom}\, L \subset X \to Z$ be a linear mapping, and let $N : X \to Z$ be a continuous mapping. If L is a Fredholm mapping of index zero and there exist continuous projectors $P : X \to X$ and $Q : Z \to Z$ such that $\mathrm{Im}P = \mathrm{Ker}\, L$, $\mathrm{Ker}\, Q = \mathrm{Im}L = \mathrm{Im}(I - Q)$, then the restriction $L|_{\mathrm{Dom}\, L \cap \mathrm{Ker}\, P} : (I - P)X \to \mathrm{Im}L$ is invertible. Denote the inverse of that map by K_P. If Ω is an open bounded subset of X, the mapping N will be called L-compact on $\overline{\Omega}$ if $QN(\overline{\Omega})$ is bounded and $K_P(I - Q)N : \overline{\Omega} \to X$ is compact. Since $\mathrm{Im}Q$ is isomorphic to $\mathrm{Ker}\, L$, there exists an isomorphism $J : \mathrm{Im}Q \to \mathrm{Ker}\, L$.

Lemma 2.5 (Continuation theorem [9]). *Let X, Z be two Banach spaces, and let L be a Fredholm mapping of index zero. Assume that $N : \overline{\Omega} \to Z$ is L-compact on $\overline{\Omega}$ with Ω being open bounded in X. Furthermore assume the following:*

(a) *for each $\lambda \in (0, 1)$, $x \in \partial\Omega \cap \mathrm{Dom}\, L$, $Lx \neq \lambda Nx$;*

(b) *for each $x \in \partial\Omega \cap \mathrm{Ker}\, L$, $QNx \neq 0$;*

(c) $\deg\{JQN, \Omega \cap \mathrm{Ker}\, L, 0\} \neq 0$.

Then the operator equation $Lx = Nx$ has at least one solution in $\mathrm{Dom}\, L \cap \overline{\Omega}$.

3. Existence of Periodic Solutions

Our main result on the existence of two positive periodic solutions for system (1.3) is stated in the following theorem.

Theorem 3.1. *Assume that the following holds:*

(i) $a^L - c_1^M > 2\sqrt{b^M h_1^M}$,

(ii) $c_2^M > d^L \alpha^L$,

(iii) $(d^L\alpha^L - c_2^M)h_1^L + h_2^L a^M > 2\sqrt{a^M d^L h_1^L \alpha^L h_2^L}$,

then system (1.3) *has at least two positive ω-periodic solutions.*

Proof. Let

$$X = Z = \left\{ z = (z_1, z_2)^{\mathrm{T}} \in C_{\mathrm{rd}}\left(\mathbb{T}, \mathbb{R}^2\right) \mid z_i(t + \omega) = z_i(t),\ i = 1, 2,\ \forall t \in \mathbb{T} \right\}, \tag{3.1}$$

and define

$$\|z\| = \left\| (z_1, z_2)^{\mathrm{T}} \right\| = \sum_{i=1}^{2} \max_{t \in I_\omega} |z_i(t)|, \quad z \in X \text{ (or } Z), \tag{3.2}$$

where $|\cdot|$ is the Euclidean norm. Then X and Z are both Banach spaces with the above norm $\|\cdot\|$. Let

$$Nz(t) = \begin{pmatrix} f_1(t) \\ f_2(t) \end{pmatrix} = \begin{pmatrix} a(t) - b(t)e^{z_1(t)} - \dfrac{c_1(t)e^{z_2(t)}}{e^{z_2(t)} + \alpha(t)e^{z_1(t)} + \beta(t)e^{2z_2(t)}} - h_1(t)e^{-z_1(t)} \\ -d(t) + \dfrac{c_2(t)e^{z_1(t)}}{e^{z_2(t)} + \alpha(t)e^{z_1(t)} + \beta(t)e^{2z_2(t)}} - h_2(t)e^{-z_2(t)} \end{pmatrix},$$
$$Lz(t) = z^{\Delta}(t), \quad Pz(t) = Qz(t) = \frac{1}{\omega}\int_{I_\omega} z(t)\Delta t, \quad z \in X. \tag{3.3}$$

Then

$$\operatorname{Ker} L = \left\{ z \in X \mid z = (z_1(t), z_2(t))^{\mathrm{T}} \in \mathbb{R}^2 \text{ for } t \in \mathbb{T} \right\},$$
$$\operatorname{Im} L = \left\{ (z_1, z_2)^{\mathrm{T}} \in Z \mid \int_{I_\omega} z_i(t)\Delta t = 0,\ i = 1,2,\ \text{for } t \in \mathbb{T} \right\}, \tag{3.4}$$

and $\dim\operatorname{Ker} L = \operatorname{codim}\operatorname{Im} L = 2$. Since $\operatorname{Im} L$ is closed in Z, then L is a Fredholm mapping of index zero. It is easy to show that P, Q are continuous projectors such that $\operatorname{Im} P = \operatorname{Ker} L$ and $\operatorname{Ker} Q = \operatorname{Im} L = \operatorname{Im}(I - Q)$. Furthermore, the generalized inverse (to L) $K_P : \operatorname{Im} L \to \operatorname{Ker} P \cap \operatorname{Dom} L$ exists and is given by

$$K_P z = K_P \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} \displaystyle\int_\kappa^t z_1(s)\Delta s - \frac{1}{\omega}\int_\kappa^{\kappa+\omega}\int_\kappa^t z_1(s)\Delta s\,\Delta t \\ \displaystyle\int_\kappa^t z_2(s)\Delta s - \frac{1}{\omega}\int_\kappa^{\kappa+\omega}\int_\kappa^t z_2(s)\Delta s\,\Delta t \end{pmatrix}. \tag{3.5}$$

Thus

$$QNz = QN\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} \displaystyle\frac{1}{\omega}\int_\kappa^{\kappa+\omega} f_1(t)\Delta t \\ \displaystyle\frac{1}{\omega}\int_\kappa^{\kappa+\omega} f_2(t)\Delta t \end{pmatrix},$$
$$K_P(I-Q)Nz = \begin{pmatrix} \displaystyle\int_\kappa^t f_1(s)\Delta s - \frac{1}{\omega}\int_\kappa^{\kappa+\omega}\int_\kappa^t f_1(s)\Delta s\,\Delta t + \left(\frac{1}{2} - \frac{t}{\omega}\right)\int_\kappa^{\kappa+\omega}(t-\kappa)f_1(s)\Delta s \\ \displaystyle\int_\kappa^t f_2(s)\Delta s - \frac{1}{\omega}\int_\kappa^{\kappa+\omega}\int_\kappa^t f_2(s)\Delta s\,\Delta t + \left(\frac{1}{2} - \frac{t}{\omega}\right)\int_\kappa^{\kappa+\omega}(t-\kappa)f_2(s)\Delta t \end{pmatrix}. \tag{3.6}$$

Obviously, $QN : X \to Z$, $K_P(I-Q)N : X \to X$ are continuous. Since X is a Banach space, using the Arzela-Ascoli theorem, it is easy to show that $\overline{K_P(I-Q)N(\Omega)}$ is compact for any open bounded set $\Omega \subset X$. Moreover, $QN(\overline{\Omega})$ is bounded, thus, N is L-compact on $\overline{\Omega}$ for any open bounded set $\Omega \subset X$. Corresponding to the operator equation $Lz = \lambda Nz$, $\lambda \in (0,1)$, we have

$$z_i^{\Delta}(t) = \lambda f_i(t), \quad i = 1, 2. \tag{3.7}$$

Suppose that $(z_1, z_2)^{\mathrm{T}} \in X$ is a solution of (3.7) for certain $\lambda \in (0,1)$. Then there exist $\xi_i, \eta_i \in I_\omega$, $i = 1, 2$, such that

$$z_i(\xi_i) = \min_{t \in I_\omega}\{z_i(t)\}, \quad z_i(\eta_i) = \max_{t \in I_\omega}\{z_i(t)\}, \quad i = 1, 2. \tag{3.8}$$

It is clear that $z_i^{\Delta}(\xi_i) = 0$, $z_i^{\Delta}(\eta_i) = 0$, $i = 1, 2$. From this and system (3.7), we have

$$f_1(\xi_1) = 0, \tag{3.9a}$$

$$f_2(\xi_2) = 0, \tag{3.9b}$$

$$f_1(\eta_1) = 0, \tag{3.10a}$$

$$f_2(\eta_2) = 0. \tag{3.10b}$$

It follows from (3.9a) that

$$z_1(\xi_1) < \ln \frac{a^M}{b^L}. \tag{3.11}$$

From (3.9b), we obtain

$$d^L \le d(\xi_2) < \frac{c_2(\xi_2)e^{z_1(\xi_2)}}{e^{z_2(\xi_2)} + \alpha(\xi_2)e^{z_1(\xi_2)} + \beta(\xi_2)e^{2z_2(\xi_2)}} < \frac{c_2^M a^M}{b^L\left(e^{z_2(\xi_2)} + \beta^L e^{2z_2(\xi_2)}\right)}, \tag{3.12}$$

which leads to

$$z_2(\xi_2) < \ln \frac{-b^L d^L + \sqrt{\left(b^L d^L\right)^2 + 4b^L d^L \beta^L c_2^M a^M}}{2b^L d^L \beta^L} =: \ln \rho. \tag{3.13}$$

Equation (3.10a) yields

$$z_1(\eta_1) > \ln \frac{h_1^L}{a^M}. \tag{3.14}$$

Equation (3.10b) deduces

$$\frac{c_2^M}{\alpha^L} > \frac{c_2(\eta_2)e^{z_1(\eta_2)}}{e^{z_2(\eta_2)} + \alpha(\eta_2)e^{z_1(\eta_2)} + \beta(\eta_2)e^{2z_2(\eta_2)}} > \frac{h_2^L}{e^{z_1(\eta_2)}}, \tag{3.15}$$

which implies

$$z_2(\eta_2) > \ln \frac{\alpha^L h_2^L}{c_2^M}. \tag{3.16}$$

From (3.9a), we also have

$$b(\xi_1)e^{2z_1(\xi_1)} + \frac{c_1(\xi_1)e^{z_1(\xi_1)+z_2(\xi_1)}}{e^{z_2(\xi_1)} + \alpha(\xi_1)e^{z_1(\xi_1)} + \beta(\xi_1)e^{2z_2(\xi_1)}} + h_1(\xi_1) - a(\xi_1)e^{z_1(\xi_1)} = 0, \tag{3.17}$$

which implies that

$$b^M e^{2z_1(\xi_1)} + \left(c_1^M - a^L\right)e^{z_1(\xi_1)} + h_1^M > 0. \tag{3.18}$$

Set

$$u_\pm = \frac{(a^L - c_1^M) \pm \sqrt{\left(c_1^M - a^L\right)^2 - 4b^M h_1^M}}{2b^M}, \tag{3.19}$$

then

$$z_1(\xi_1) > \ln u_+ \quad \text{or} \quad z_1(\xi_1) < \ln u_-. \tag{3.20}$$

From (3.10a), a parallel argument to (3.20) gives

$$z_1(\eta_1) > \ln u_+ \quad \text{or} \quad z_1(\eta_1) < \ln u_-. \tag{3.21}$$

From (3.9b), we also obtain

$$\frac{c_2(\xi_2)e^{z_1(\xi_2)}}{e^{z_2(\xi_2)} + \alpha(\xi_2)e^{z_1(\xi_2)}} - d(\xi_2) - h_2(\xi_2)e^{-z_2(\xi_2)} \geq 0, \tag{3.22}$$

which implies

$$a^M d^L e^{2z_2(\xi_2)} + \left[\left(d^L\alpha^L - c_2^M\right)h_1^L + h_2^L a^M\right]e^{z_2(\xi_2)} + h_1^L h_2^L \alpha^L \leq 0. \tag{3.23}$$

Set

$$v_\pm = \frac{(c_2^M - d^L\alpha^L)h_1^L - h_2^L a^M \pm \sqrt{\left((c_2^M - d^L\alpha^L)h_1^L - h_2^L a^M\right)^2 - 4a^M d^L h_1^L h_2^L \alpha^L}}{2a^M d^L}, \tag{3.24}$$

then

$$\ln v_- < z_2(\xi_2) < \ln v_+ \quad \text{or} \quad \ln v_+ < z_2(\eta_2) < \ln v_-. \tag{3.25}$$

From (3.11), (3.14), (3.20), and (3.21), we obtain for all $t \in I_\omega$,

$$\ln \frac{h_1^L}{a^M} < z_1(t) < \ln u_- \tag{3.26}$$

or

$$\ln u_+ < z_1(t) < \ln \frac{a^M}{b^L}. \tag{3.27}$$

From (3.13), (3.16), and (3.25), we obtain that for all $t \in I_\omega$,

$$A := \max\left\{\ln v_-, \ln \frac{\alpha^L h_2^L}{c_2^M}\right\} < z_2(t) < \min\{\ln v_+, \ln \rho\} =: B. \tag{3.28}$$

Obviously, $\ln(h_1^L/a^M)$, $\ln(a^M/b^L)$, $\ln u_\pm$, A, B are independent of λ. Now take

$$\begin{aligned}
\Omega_1 &= \left\{z = (z_1, z_2)^{\mathrm{T}} \in X \mid z_1(t) \in \left(\ln \frac{h_1^L}{a^M}, \ln u_-\right),\ z_2(t) \in (A, B)\right\},\\
\Omega_2 &= \left\{z = (z_1, z_2)^{\mathrm{T}} \in X \mid z_1(t) \in \left(\ln u_+, \ln \frac{a^M}{b^L}\right),\ z_2(t) \in (A, B)\right\},
\end{aligned} \tag{3.29}$$

then Ω_1 and Ω_2 are bounded open subsets of X, and $\Omega_1 \cap \Omega_2 = \emptyset$. Thus Ω_1 and Ω_2 satisfy the requirement (a) in Lemma 2.5.

Now we show that (b) of Lemma 2.5 holds; that is, we need to prove when $z = (z_1, z_2)^{\mathrm{T}} \in \partial\Omega_i \cap \operatorname{Ker} L = \partial\Omega_i \cap \mathbb{R}^2$, $QNz \neq (0,0)^{\mathrm{T}}$, $i = 1,2$. If it is not true, then when $z \in \partial\Omega_i \cap \operatorname{Ker} L = \partial\Omega_i \cap \mathbb{R}^2$, $i = 1,2$, constant vector z with $z \in \partial\Omega_i$, $i = 1,2$, satisfies $QNz = (0,0)^{\mathrm{T}}$. Thus there exist two points $t_i \in I_\omega$ $(i = 1,2)$ such that

$$\begin{aligned}
a(t_1) - b(t_1)e^{z_1} - \frac{c_1(t_1)e^{z_2}}{e^{z_2} + \alpha(t_1)e^{z_1} + \beta(t_1)e^{2z_2}} - h_1(t_1)e^{-z_1} &= 0,\\
-d(t_2) + \frac{c_2(t_2)e^{z_1}}{e^{z_2} + \alpha(t_2)e^{z_1} + \beta(t_2)e^{2z_2}} - h_2(t_2)e^{-z_2} &= 0.
\end{aligned} \tag{3.30}$$

From this and following the arguments of (3.26)–(3.28), we have

$$\ln \frac{h_1^L}{a^M} < z_1 < \ln u_-, \quad A < z_2 < B \quad \text{or} \quad \ln u_+ < z_1 < \ln \frac{a^M}{b^L}, \quad A < z_2 < B. \tag{3.31}$$

Thus $z \in \Omega_1 \cap \mathbb{R}^2$, or $z \in \Omega_2 \cap \mathbb{R}^2$. This contradicts the fact that $z \in \partial\Omega_i \cap \mathbb{R}^2$, $i = 1,2$. Hence condition (b) in Lemma 2.5 holds.

Finally, we show that (c) in Lemma 2.5 is valid. Noticing that the system of algebraic equations

$$\begin{aligned} a(t_1) - b(t_1)\mathrm{e}^x - \frac{h_1(t_1)}{\mathrm{e}^x} &= 0, \\ -d(t_2) + \frac{c_2(t_2)\mathrm{e}^x}{\mathrm{e}^y + \alpha(t_2)\mathrm{e}^x + \beta(t_2)\mathrm{e}^{2y}} - \frac{h_2(t_2)}{\mathrm{e}^y} &= 0 \end{aligned} \tag{3.32}$$

has two distinct solutions (x_1^*, y_1^*) and (x_2^*, y_2^*) since the conditions hold in Theorem 3.1, and we have

$$(x_1^*, y_1^*) = (\ln x_-, \ln y_-), \qquad (x_2^*, y_2^*) = (\ln x_+, \ln y_+), \tag{3.33}$$

where

$$\begin{aligned} x_\pm &= \frac{a(t_1) \pm \sqrt{(a(t_1))^2 - 4b(t_1)h_1(t_1)}}{2b(t_1)}, \\ y_\pm &= \frac{1}{2a(t_2)d(t_2)}\Big\{(c_2(t_2) - d(t_2)\alpha(t_2))h_1(t_2) - h_2(t_2)a(t_2) \\ &\qquad \pm\sqrt{((c_2(t_2)-d(t_2)\alpha(t_2))h_1(t_2)-h_2(t_2)a(t_2))^2-4a(t_2)d(t_2)h_1(t_2)h_2(t_2)\alpha(t_2)}\Big\}. \end{aligned} \tag{3.34}$$

It is easy to verify that

$$\begin{aligned} &\ln\frac{h_1^L}{a^M} < \ln x_- < \ln u_- < \ln u_+ < \ln x_+ < \ln\frac{a^M}{b^L}, \\ &\ln\frac{\alpha^L h_1^L}{c_2^M} < \ln y_- < \ln v_- < \ln v_+ < \ln y_+ < \ln \rho. \end{aligned} \tag{3.35}$$

Therefore, $(x_1^*, y_1^*) \in \Omega_1$, $(x_2^*, y_2^*) \in \Omega_2$.

Define the homotopy $H_\mu(z_1, z_2) : \operatorname{Dom} L \times [0,1] \to \mathbb{R}^2$ by

$$H_\mu(z_1, z_2) = \mu QN(z_1, z_2) + (1-\mu)G(z_1, z_2), \quad \text{for } \mu \in [0,1], \tag{3.36}$$

where

$$G\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} a(t_1) - b(t_1)u - \dfrac{h_1(t_1)}{u} \\ -d(t_2) + \dfrac{c_2(t_2)u}{v + \alpha(t_2)u + \beta(t_2)v^2} - \dfrac{h_2(t_2)}{v} \end{pmatrix}, \tag{3.37}$$

and $\mu \in [0,1]$ is a parameter, $u = e^{z_1}$, $v = e^{z_2}$, and $(u,v)^{\mathrm{T}}$ is a constant vector in $\mathbb{R}^2$. It is easy to show that $0 \notin H_\mu(\partial\Omega_i \cap \mathrm{Ker}L)$ $(i = 1,2)$. In fact, if there are a certain $z^* = (z_1^*, z_2^*)$ and a certain $\mu^* \in [0,1]$ such that $H_{\mu^*}(z_1^*, z_2^*) = 0$, where $z_i^* \in \partial\Omega_i \cap \mathrm{Ker}L$ $(i = 1,2)$, then there exist $t_i^* \in I_\omega$ $(i = 1,2)$ such that

$$\begin{gathered}
a(t_1^*) - b(t_1^*)e^{z_1^*} - \frac{\mu^* c_1(t_1^*)e^{z_2^*}}{e^{z_2^*} + \alpha(t_1^*)e^{z_1^*} + \beta(t_1^*)e^{2z_2^*}} - h_1(t_1^*)e^{-z_1^*} = 0, \\
-d(t_2^*) + \frac{c_2(t_2^*)e^{z_1^*}}{e^{z_2^*} + \alpha(t_2^*)e^{z_1^*} + \beta(t_2^*)e^{2z_2^*}} - h_2(t_2^*)e^{-z_2^*} = 0.
\end{gathered} \tag{3.38}$$

By carrying out similar arguments as above, we also obtain conclusions as same as (3.26)–(3.28), that is,

$$\ln \frac{h_1^L}{a^M} < z_1^* < \ln u_-, \quad A < z_2^* < B \quad \text{or} \quad \ln u_+ < z_1^* < \ln \frac{a^M}{b^L}, \quad A < z_2^* < B. \tag{3.39}$$

This contradicts the fact that $z_i^* \in \partial\Omega_i \cap \mathrm{Ker}L$ $(i = 1,2)$.

Note that $J = I$ (identical mapping), since $\mathrm{Im}Q = \mathrm{Ker}\,L$, according to the invariance property of homotopy, direct calculation produces

$$\begin{aligned}
&\deg\left\{JQN(z_1,z_2)^{\mathrm{T}}, \Omega_i \cap \mathrm{Ker}\,L, (0,0)^{\mathrm{T}}\right\} \\
&\quad= \deg\left\{G(z_1,z_2)^{\mathrm{T}}, \Omega_i \cap \mathrm{Ker}\,L, (0,0)^{\mathrm{T}}\right\} \\
&\quad= \mathrm{sign}\begin{vmatrix} -b(t_1) + \dfrac{h_1(t_1)}{u_i^{*2}} & 0 \\ \dfrac{c_2(t_2)\left(v_i^* + \beta(t_2)v_i^{*2}\right)}{\left(v_i^* + \alpha(t_2)u_i^* + \beta(t_2)v_i^{*2}\right)^2} & \dfrac{h_2(t_2)}{v_i^{*2}} - \dfrac{c_2(t_2)u_i^*(1+2\beta(t_2)v_i^*)}{\left(v_i^* + \alpha(t_2)u_i^* + \beta(t_2)v_i^{*2}\right)^2} \end{vmatrix} \\
&\quad= \mathrm{sign}\begin{vmatrix} -b(t_1)u_i^* + \dfrac{h_1(t_1)}{u_i^*} & 0 \\ \dfrac{c_2(t_2)v_i^*\left(v_i^* + \beta(t_2)v_i^{*2}\right)}{\left(v_i^* + \alpha(t_2)u_i^* + \beta(t_2)v_i^{*2}\right)^2} & \dfrac{h_2(t_2)}{v_i^*} - \dfrac{c_2(t_2)u_i^*v_i^*(1+2\beta(t_2)v_i^*)}{\left(v_i^* + \alpha(t_2)u_i^* + \beta(t_2)v_i^{*2}\right)^2} \end{vmatrix} \\
&\quad= \mathrm{sign}\left[\left(-b(t_1)u_i^* + \frac{h_1(t_1)}{u_i^*}\right)\left(\frac{h_2(t_2)}{v_i^*} - \frac{c_2(t_2)u_i^*v_i^*(1+2\beta(t_2)v_i^*)}{\left(v_i^* + \alpha(t_2)u_i^* + \beta(t_2)v_i^{*2}\right)^2}\right)\right] \\
&\quad= \mathrm{sign}\left[(a(t_1) - 2b(t_1)u_i^*)\left(\frac{h_2(t_2)}{v_i^*} - \frac{(v_i^*d(t_2) + h_2(t_2))(1+2\beta(t_2)v_i^*)}{c_2(t_2)u_i^*}\left(\frac{h_2(t_2)}{v_i^*} + d(t_2)\right)\right)\right],
\end{aligned} \tag{3.40}$$

where $\deg\{\cdot,\cdot,\cdot\}$ is the Brouwer degree, and u_i^*, v_i^* are positive solution for (3.32) in Ω_i, respectively. And

$$\frac{h_2(t_2)}{v_i^*} - \frac{(v_i^* d(t_2) + h_2(t_2))(1 + 2\beta(t_2)v_i^*)}{c_2(t_2)u_i^*}\left(\frac{h_2(t_2)}{v_i^*} + d(t_2)\right) \neq 0, \tag{3.41}$$

otherwise, we have $c_2(t_2)u_i^* h_2(t_2) = (1+2\beta(t_2)v_i^*)(v_i^* d(t_2)+h_2(t_2))^2$. After test and verification, it is not possible. Thus

$$\begin{aligned}
&\deg\left\{JQN(z_1,z_2)^{\mathrm{T}}, \Omega_1 \cap \operatorname{Ker} L, (0,0)^{\mathrm{T}}\right\} \\
&\quad = \deg\left\{G(z_1,z_2)^{\mathrm{T}}, \Omega_1 \cap \operatorname{Ker} L, (0,0)^{\mathrm{T}}\right\} \\
&\quad = \operatorname{sign}\left[(a(t_1) - 2b(t_1)x_-)\left(\frac{h_2(t_2)}{y_-} - \frac{(y_- d(t_2) + h_2(t_2))(1 + 2\beta(t_2)y_-)}{c_2(t_2)x_-}\left(\frac{h_2(t_2)}{y_-} + d(t_2)\right)\right)\right] \\
&\quad = -1, \\
&\deg\left\{JQN(z_1,z_2)^{\mathrm{T}}, \Omega_2 \cap \operatorname{Ker} L, (0,0)^{\mathrm{T}}\right\} = \deg\left\{G(z_1,z_2)^{\mathrm{T}}, \Omega_2 \cap \operatorname{Ker} L, (0,0)^{\mathrm{T}}\right\} \\
&\quad = \operatorname{sign}\left[(a(t_1) - 2b(t_1)x_+)\left(\frac{h_2(t_2)}{y_+} - \frac{(y_+ d(t_2) + h_2(t_2))(1 + 2\beta(t_2)y_+)}{c_2(t_2)x_+}\left(\frac{h_2(t_2)}{y_+} + d(t_2)\right)\right)\right] \\
&\quad = 1.
\end{aligned} \tag{3.42}$$

By now we have proved that Ω_i $(i = 1,2)$ verifies all requirements of Lemma 2.5. Therefore, system (1.3) has at least two ω-periodic solutions in $\operatorname{Dom} L \cap \overline{\Omega_i}$ $(i = 1,2)$, respectively. The proof is complete. □

Corollary 3.2. *If the conditions in Theorem 3.1 hold, then both the corresponding continuous model* (1.4) *and the discrete model* (1.5) *have at least two ω-periodic solutions.*

Remark 3.3. If $\beta(t) \equiv 0$ in system (1.3), then the system is a ratio-dependent predator-prey model with Holling-type functional response and harvesting terms on time scales:

$$\begin{aligned}
z_1^{\Delta}(t) &= a(t) - b(t)e^{z_1(t)} - \frac{c_1(t)e^{z_2(t)}}{e^{z_2(t)} + \alpha(t)e^{z_1(t)}} - h_1(t)e^{-z_1(t)}, \\
z_2^{\Delta}(t) &= -d(t) + \frac{c_2(t)e^{z_1(t)}}{e^{z_2(t)} + \alpha(t)e^{z_1(t)}} - h_2(t)e^{-z_2(t)}.
\end{aligned} \tag{3.43}$$

Therefore, we have the following results.

Corollary 3.4. *If the conditions in Theorem 3.1 hold, then system* (3.43) *has at least two ω-periodic solutions. Specifically, both the corresponding continuous model and the discrete model of the system* (3.43) *have at least two ω-periodic solutions.*

Remark 3.5. If $h_1(t) \equiv 0$, $h_2(t) \equiv 0$ in system (1.3), then the system is a general ratio-dependent predator-prey model with Beddington-DeAngelis functional response on time scales:

$$\begin{aligned} z_1^{\Delta}(t) &= a(t) - b(t)e^{z_1(t)} - \frac{c_1(t)e^{z_2(t)}}{e^{z_2(t)} + \alpha(t)e^{z_1(t)} + \beta(t)e^{2z_2(t)}}, \\ z_2^{\Delta}(t) &= -d(t) + \frac{c_2(t)e^{z_1(t)}}{e^{z_2(t)} + \alpha(t)e^{z_1(t)} + \beta(t)e^{2z_2(t)}}. \end{aligned} \tag{3.44}$$

Using Lemma 2.5, we can obtain another important conclusion as follows.

Theorem 3.6. *Assume that the following holds:*

(i) $a^L > c_1^M$,

(ii) $c_2^M > d^L \alpha^L$,

(iii) $a^L > c_1^M / \beta^L$,

then system (3.44) *has at least a positive* ω*-periodic solutions.*

Corollary 3.7. *If the conditions in Theorem 3.6 hold, then both the corresponding continuous model and the discrete model of the system* (3.44) *have at least a positive* ω*-periodic solutions.*

Acknowledgments

The author is very grateful to his supervisor Professor M. Fan for his many valuable comments and suggestions which greatly improved the presentation of this paper. In addition, he would like to express his gratitude to the reviewers and editor for their valuable comments and careful reading and the time and efforts he/she has spent in the review. This work is supported by the Foundation for Subjects Development of Harbin University (no. HXK200716).

References

[1] Z. Zhang and H. Tang, "Four positive periodic solutions for the first order differential system," *Journal of Mathematical Analysis and Applications*, vol. 332, no. 1, pp. 123–136, 2007.

[2] D. Hu and Z. Zhang, "Four positive periodic solutions of a discrete time delayed predator-prey system with nonmonotonic functional response and harvesting," *Computers & Mathematics with Applications*, vol. 56, no. 12, pp. 3015–3022, 2008.

[3] H. Fang and Y. Xiao, "Existence of multiple periodic solutions for delay Lotka-Volterra competition patch systems with harvesting," *Applied Mathematical Modelling*, vol. 33, no. 2, pp. 1086–1096, 2009.

[4] Z. Zhang and T. Tian, "Multiple positive periodic solutions for a generalized predator-prey system with exploited terms," *Nonlinear Analysis: Real World Applications*, vol. 9, no. 1, pp. 26–39, 2008.

[5] Z. Ma, *Mathematical Modeling and Studying on Species Ecology*, Education Press, Hefei, China, 1996.

[6] Z. Zhang and Z. Hou, "Existence of four positive periodic solutions for a ratio-dependent predator-prey system with multiple exploited (or harvesting) terms," *Nonlinear Analysis: Real World Applications*. In press.

[7] M. Bohner and A. Peterson, *Dynamic Equations on Time Scales: An Introduction with Application*, Birkhäuser, Boston, Mass, USA, 2001.

[8] E. R. Kaufmann and Y. N. Raffoul, "Periodic solutions for a neutral nonlinear dynamical equation on a time scale," *Journal of Mathematical Analysis and Applications*, vol. 319, no. 1, pp. 315–325, 2006.

[9] R. E. Gaines and J. L. Mawhin, *Coincidence Degree, and Nonlinear Differential Equations*, vol. 568 of *Lecture Notes in Mathematics*, Springer, Berlin, Germany, 1977.