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Research Article

A New Part-Metric-Related Inequality Chain and an Application

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Part-metric-related (PMR) inequality chains are elegant and are useful in the study of difference equations. In this paper, we establish a new PMR inequality chain, which is then applied to show the global asymptotic stability of a class of rational difference equations.

1. Introduction

A part-metric related (PMR) inequality chain is a chain of inequalities of the form

$$\min_{1\le i\le n}\left\{a_i, \frac{1}{a_i}\right\} \le f(a_1,\dots,a_n) \le \max_{1\le i\le n}\left\{a_i, \frac{1}{a_i}\right\}, \tag{1.1}$$

which is closely related to the well-known part metric [1] and has important applications in the study of difference equations [2–13]. Below are three previously known PMR inequality chains:

$$\min_{1\le i\le 4}\left\{a_i, \frac{1}{a_i}\right\} \le \frac{a_1+a_2+a_3a_4}{a_1a_2+a_3+a_4} \le \max_{1\le i\le 4}\left\{a_i, \frac{1}{a_i}\right\} \quad (\text{see [5]}), \tag{1.2}$$

$$\min_{1\le i\le k}\left\{a_i, \frac{1}{a_i}\right\} \le \frac{a_1+\cdots+a_{k-2}+a_{k-1}a_k}{a_1a_2+a_3+\cdots+a_k} \le \max_{1\le i\le k}\left\{a_i, \frac{1}{a_i}\right\} \quad (\text{see [11]}), \tag{1.3}$$

$$\min_{1\le i\le 5}\left\{a_i, \frac{1}{a_i}\right\} \le \frac{(1+w)a_1a_2a_3+a_4+a_5}{a_1a_2+a_1a_3+a_2a_3+wa_4a_5} \le \max_{1\le i\le 5}\left\{a_i, \frac{1}{a_i}\right\}, \quad 1\le w\le 2 \quad (\text{see [13]}). \tag{1.4}$$

In this article, we establish the following PMR inequality chain:

$$\min_{1\le i\le 2p-1}\left\{a_i,\frac{1}{a_i}\right\}\le h_w(a_1,\dots,a_{2p-1})\le\max_{1\le i\le 2p-1}\left\{a_i,\frac{1}{a_i}\right\},\tag{1.5}$$

where h_w will be defined in the next section, $p-2\le w\le p-1$. When $p=3$, chain (1.5) reduces to chain (1.4). On this basis, we prove that the difference equation

$$x_n=h_w(x_{n-2p+1},\dots,x_{n-1}),\quad n=1,2,\dots,\tag{1.6}$$

with positive initial conditions admits a globally asymptotically stable equilibrium $c=1$.

2. Main results

This section establishes the main results of this paper. For a function $f(a_1,\dots,a_{2p-1})$, let

$$f(a_1,\dots,a_{2p-1})|_{i_1\sim i_r}=f(a_1,\dots,a_{2p-1})|_{a_{i_j}=m,1\le j\le r}.\tag{2.1}$$

Lemma 2.1. *Let $a_1,\dots,a_n,b_1,\dots,b_n>0$. Then $\min_{1\le i\le n}\{a_i/b_i\}\le(a_1+\cdots+a_n)/(b_1+\cdots+b_n)\le\max_{1\le i\le n}\{a_i/b_i\}$. One equality in the chain holds if and only if $a_1/b_1=\cdots=a_n/b_n$.*

For $p\ge 3$ and $w>0$, define a function $h_w:(\Re_+)^{2p-1}\to\Re_+$ as follows:

$$h_w(a_1,\dots,a_{2p-1})=\frac{(1+w)\prod_{i=1}^{p}a_i+\prod_{i=p+1}^{2p-1}a_i\times\sum_{i=p+1}^{2p-1}(1/a_i)}{\prod_{i=1}^{p}a_i\times\sum_{i=1}^{p}(1/a_i)+w\prod_{i=p+1}^{2p-1}a_i}.\tag{2.2}$$

Below are two examples of this function:

$$\begin{aligned}h_w(a_1,\dots,a_5)&=\frac{(1+w)a_1a_2a_3+a_4+a_5}{a_1a_2+a_1a_3+a_2a_3+wa_4a_5},\\ h_w(a_1,\dots,a_7)&=\frac{(1+w)a_1a_2a_3a_4+a_5a_6+a_5a_7+a_6a_7}{a_1a_2a_3+a_1a_2a_4+a_1a_3a_4+a_2a_3a_4+wa_5a_6a_7}.\end{aligned}\tag{2.3}$$

For brevity, let $h_w=h_w(a_1,\dots,a_{2p-1})$. Note that, for each a_r, h_w is linear fractional in a_r. As a consequence, h_w is monotone in a_r. Through simple calculations, we get the following two lemmas.

Lemma 2.2. *Let $p \geq 3$, $a_1, \ldots, a_{2p-1} > 0$, $m = \min_{1\leq i\leq 2p-1}\{a_i\}$, $1 \leq r \leq p$.*

(1) *If h_{p-2} is increasing in a_r, then $h_{p-2} \leq (p-1)/\sum_{i=1,i\neq r}^{p}(1/a_i)$. The equality holds if and only if h_{p-2} is constant in a_r.*

(2) *If h_{p-2} is strictly decreasing in a_r, then $h_{p-2} \leq h_{p-2}|_{a_r=m}$. The equality holds if and only if $a_r = m$.*

Lemma 2.3. *Let $p \geq 3$, $a_1, \ldots, a_{2p-1} > 0$, $m = \min_{1\leq i\leq 2p-1}\{a_i\}$, $p+1 \leq r \leq 2p-1$.*

(1) *If h_{p-2} is increasing in a_r, then $h_{p-2} \leq \sum_{i=p+1,i\neq r}^{2p-1}(1/a_i)/(p-2)$. The equality holds if and only if h_{p-2} is constant in a_r.*

(2) *If h_{p-2} is strictly decreasing in a_r, then $h_{p-2} \leq h_{p-2}|_{a_r=m}$. The equality holds if and only if $a_r = m$.*

Theorem 2.4. *Let $p \geq 3$, $a_1, \ldots, a_{2p-1} > 0$. Then $\min_{1\leq i\leq 2p-1}\{a_i, 1/a_i\} \leq h_{p-2} \leq \max_{1\leq i\leq 2p-1}\{a_i, 1/a_i\}$. One of the two equalities holds if and only if $a_1 = \cdots = a_{2p-1} = 1$.*

Proof. Let $m = \min_{1\leq i\leq 2p-1}\{a_i\}$, $M = \max_{1\leq i\leq 2p-1}\{a_i\}$.

We prove only $h_{p-2} \leq \max\{M, 1/m\}$ because $\min\{M, 1/m\} \leq h_{p-2}$ can be proved similarly. We proceed by distinguishing two possible cases.

Case 1. There is a permutation $i_1, \ldots, i_{2p-1}$ of $1, 2, \ldots, 2p-1$ such that for each $1 \leq k \leq 2p-1$, either $a_{i_k} = m$ or $h_{p-2}|_{i_1\sim i_{k-1}}$ is strictly decreasing in a_{i_k}. Then

$$h_{p-2} \leq h_{p-2}|_{i_1} \leq \cdots \leq h_{p-2}|_{i_1\sim i_{2p-1}} = \frac{1}{2}\left(m + \frac{1}{m}\right) \leq \max\left\{m, \frac{1}{m}\right\} \leq \max\left\{M, \frac{1}{m}\right\}. \tag{2.4}$$

Case 2. There is a partial permutation $i_1, \ldots, i_r$ of $1, 2, \ldots, 2p-1$ $(1 \leq r \leq 2p-2)$ such that (a) for each $1 \leq k \leq r$, either $a_{i_k} = m$ or $h_{p-2}|_{i_1\sim i_{k-1}}$ is strictly decreasing in a_{i_k}, and (b) for each $t \in \{1, \ldots, 2p-1\} - \{i_1, \ldots, i_r\}$, $a_{i_t} \neq m$ and $h_{p-2}|_{i_1\sim i_r}$ is increasing in a_t. Then

$$m < M, \qquad h_{p-2} \leq h_{p-2}|_{i_1} \leq h_{p-2}|_{i_1\sim i_2} \leq \cdots \leq h_{p-2}|_{i_1\sim i_r}. \tag{2.5}$$

Since $r \leq 2p-2$, there is $t \in \{1, \ldots, 2p-1\} - \{i_1, \ldots, i_r\}$. If $t \in \{1, \ldots, p\} - \{i_1, \ldots, i_r\}$, it follows from (2.5) and Lemma 2.2 that

$$h_{p-2} \leq h_{p-2}|_{i_1\sim i_r} \leq \frac{(p-1)}{\sum_{i=1,i\neq t}^{p}(1/a_i)|_{i_1\sim i_r}} \leq \max_{1\leq i\leq p, i\neq t}\{a_i\}|_{i_1\sim i_r} \leq M \leq \max\left\{M, \frac{1}{m}\right\}. \tag{2.6}$$

Whereas if $t \in \{p+1, \ldots, 2p-1\} - \{i_1, \ldots, i_r\}$, it follows from (2.5) and Lemma 2.3 that

$$h_{p-2} \leq h_{p-2}|_{i_1\sim i_r} \leq \frac{\sum_{i=p+1,i\neq t}^{2p-1}(1/a_i)}{(p-2)|_{i_1\sim i_r}} \leq \max_{p+1\leq i\leq 2p-1, i\neq t}\left\{\frac{1}{a_i}\right\}|_{i_1\sim i_r} \leq \frac{1}{m} \leq \max\left\{M, \frac{1}{m}\right\}. \tag{2.7}$$

Hence, $h_{p-2} \leq \max\{M, 1/m\}$ is proven.

Second, we prove that $a_1 = \cdots = a_{2p-1} = 1$ if $h_{p-2} = \max\{M, 1/m\}$. The claim of "$a_1 = \cdots = a_{2p-1} = 1$ if $h_{p-2} = \min\{M, 1/m\}$" can be treated similarly. To this end, we need to prove the following. □

Claim 1. If $h_{p-2} = \max\{M, 1/m\}$, then there is a permutation $i_1, \ldots, i_{2p-1}$ of $1, \ldots, 2p-1$ such that for each $1 \leq k \leq 2p-1$, either $a_{i_k} = m$ or $h_{p-2}|_{i_1\sim i_{k-1}}$ is strictly decreasing in a_{i_k}.

Proof of Claim 1. On the contrary, assume that Claim 1 is not true. Then there is a partial permutation $i_1, \ldots, i_r$ of $1, 2, \ldots, 2p-1$ $(1 \le r \le 2p-2)$ such that (a) for each $1 \le k \le r$, either $a_{i_k} = m$ or $h_{p-2}|_{i_1 \sim i_{k-1}}$ is strictly decreasing in a_{i_k}, and (b) for each $t \in \{1, \ldots, 2p-1\} - \{i_1, \ldots, i_r\}$, $a_{i_t} \ne m$ and $h_{p-2}|_{i_1 \sim i_r}$ is increasing in a_t. One of the following two cases must occur.

Case 1. There is $t \in \{1, \ldots, 2p-1\} - \{i_1, \ldots, i_r\}$ such that $h_{p-2}|_{i_1 \sim i_r}$ is strictly increasing in a_t. If $t \in \{1, \ldots, p\} - \{i_1, \ldots, i_r\}$, it follows by (2.5), (2.6), and Lemma 2.2 that

$$h_{p-2} \le h_{p-2}|_{i_1 \sim i_r} < \frac{(p-1)}{\sum_{i=1, i\ne t}^{p} (1/a_i)|_{i_1 \sim i_r}} \le \max_{1 \le i \le p, i \ne t} \{a_i\}|_{i_1 \sim i_r} \le \max\left\{M, \frac{1}{m}\right\}. \tag{2.8}$$

A contradiction occurs. Whereas if $t \in \{p+1, \ldots, 2p-1\} - \{i_1, \ldots, i_r\}$, it follows by (2.5), (2.7), and Lemma 2.3 that

$$h_{p-2} \le h_{p-2}|_{i_1 \sim i_r} < \frac{\sum_{i=p+1, i\ne t}^{2p-1} (1/a_i)}{(p-2)|_{i_1 \sim i_r}} \le \max_{p+1 \le i \le 2p-1, i \ne t} \left\{\frac{1}{a_i}\right\}|_{i_1 \sim i_r} \le \max\left\{M, \frac{1}{m}\right\}. \tag{2.9}$$

Again a contradiction occurs.

Case 2. For each $t \in \{1, \ldots, 2p-1\} - \{i_1, \ldots, i_r\}$, $h_{p-2}|_{i_1 \sim i_r}$ is constant in a_t.

First, let us show that $\{1, \ldots, p\} \subseteq \{i_1, \ldots, i_r\}$. Otherwise, there is $t \in \{1, \ldots, p\} - \{i_1, \ldots, i_r\}$. By Lemma 2.2, we have

$$h_{p-2}|_{i_1 \sim i_r} = \frac{(p-1)}{\sum_{i=1, i\ne t}^{p} (1/a_i)|_{i_1 \sim i_r}}. \tag{2.10}$$

If there is $s \in \{1, \ldots, p\} - \{i_1, \ldots, i_r, t\}$, it follows from (2.10) that $h_{p-2}|_{i_1 \sim i_r}$ is strictly increasing in a_s, a contradiction occurs. So, $\{1, \ldots, p\} - \{i_1, \ldots, i_r\} = \{t\}$ and thus

$$\max\left\{M, \frac{1}{m}\right\} = h_{p-2} \le h_{p-2}|_{i_1 \sim i_r} = h_{p-2}(a_1, \ldots, a_{2p-1})|_{a_{i_1} = \cdots = a_{i_r} = m} = m < M, \tag{2.11}$$

from which a contradiction follows. So, $\{1, \ldots, p\} \subseteq \{i_1, \ldots, i_r\}$. □

According to the previous argument, there is $t \in \{p+1, \ldots, 2p-1\} - \{i_1, \ldots, i_r\}$. By Lemma 2.3, we get

$$h_{p-2}|_{i_1 \sim i_r} = \frac{\sum_{i=p+1, i\ne t}^{2p-1} (1/a_i)}{(p-2)|_{i_1 \sim i_r}}. \tag{2.12}$$

If there is $s \in \{p+1, \ldots, 2p-1\} - \{i_1, \ldots, i_r, t\}$, it follows from (2.12) that $h_{p-2}|_{i_1 \sim i_r}$ is strictly decreasing in a_s, a contradiction. So, $\{p+1, \ldots, 2p-1\} - \{i_1, \ldots, i_r\} = \{t\}$ and thus

$$a_1 = \cdots = a_{t-1} = a_{t+1} = \cdots = a_{2p-1} = m. \tag{2.13}$$

By (2.13) and (2.2), we get

$$h_{p-2} = h_{p-2}|_{i_1 \sim i_r} = \frac{(p-1)m^3 + m + (p-2)a_t}{pm^2 + (p-2)ma_t}. \tag{2.14}$$

Since $h_{p-2}|_{i_1\sim i_r}$ is constant in a_t, and $(\mathrm{d}/\mathrm{d}a_t)h_{p-2}|_{i_1\sim i_r} = ((p-1)(p-2)m^2(1-m^2))/[pm^2+(p-2)ma_t]^2$, we derive $m = 1$. From (2.12) and (2.13), we get $h_{p-2}|_{i_1\sim i_r} = 1/m$. Since $h_{p-2} = \max\{M, 1/m\}$, all equalities in chains (2.5) and (2.7) hold. These plus $m = 1$ yield $h_{p-2}|_{i_1\sim i_r} = 1/m = 1 \geq M$, from which we derive $M = m = 1$. So, $a_t = 1 = m$. This is a contradiction. Claim 1 is proved.

By Claim 1 and $h_{p-2} = \max\{M, 1/m\}$, all equalities in (2.4) must hold. This plus Lemma 2.2 yields $a_1 = \cdots = a_{2p-1} = m$ and $h_{p-2}(m, \ldots, m) = (m + 1/m)/2 = m$. This implies $a_1 = \cdots = a_{2p-1} = 1$.

Theorem 2.5. *Let $p \geq 3$, $a_1, \ldots, a_{2p-1} > 0$. Then, $\min_{1\leq i\leq 2p-1}\{a_i, 1/a_i\} \leq h_{p-1} \leq \max_{1\leq i\leq 2p-1}\{a_i, 1/a_i\}$. One of the two equalities holds if and only if $a_1 = \cdots = a_p = 1/a_{p+1} = \cdots = 1/a_{2p-1}$.*

Proof. By Lemma 2.1 and (2.2), we get

$$\begin{aligned} h_{p-1} &\leq \max\left\{a_1, \ldots, a_p, \frac{1}{a_{p+1}}, \ldots, \frac{1}{a_{2p-1}}\right\} \leq \max_{1\leq i\leq 2p-1}\left\{a_i, \frac{1}{a_i}\right\}, \\ h_{p-1} &\geq \min\left\{a_1, \ldots, a_p, \frac{1}{a_{p+1}}, \ldots, \frac{1}{a_{2p-1}}\right\} \geq \min_{1\leq i\leq 2p-1}\left\{a_i, \frac{1}{a_i}\right\}. \end{aligned} \tag{2.15}$$

The second claim follows immediately from Lemma 2.1.

We are ready to present the main result of this paper. □

Theorem 2.6. *Let $p \geq 3$, $p-2 \leq w \leq p-1$, $a_1, \ldots, a_{2p-1} > 0$. Let*

$$a_k = h_w(a_{k-2p+1}, \ldots, a_{k-1}), \quad k = 2p, 2p+1, \ldots. \tag{2.16}$$

Then $\min_{1\leq i\leq 2p-1}\{a_i, 1/a_i\} \leq a_k \leq \max_{1\leq i\leq 2p-1}\{a_i, 1/a_i\}$, $k = 2p, 2p+1, \ldots$. If $k \geq 2p+1$, then one of the two equalities holds if and only if $a_1 = \cdots = a_{2p-1} = 1$.

Proof. Regard h_w as a linear fractional function in w, which is monotone in w. By Theorems 2.4 and 2.5, we obtain

$$\begin{aligned} a_{2p} &\geq \min\{h_{p-2}(a_1, \ldots, a_{2p-1}), h_{p-1}(a_1, \ldots, a_{2p-1})\} \geq \min_{1\leq i\leq 2p-1}\left\{a_i, \frac{1}{a_i}\right\}, \\ a_{2p} &\leq \max\{h_{p-2}(a_1, \ldots, a_{2p-1}), h_{p-1}(a_1, \ldots, a_{2p-1})\} \leq \max_{1\leq i\leq 2p-1}\left\{a_i, \frac{1}{a_i}\right\}, \\ a_{2p+1} &\geq \min\{h_{p-2}(a_2, \ldots, a_{2p}), h_{p-1}(a_2, \ldots, a_{2p})\} \geq \min_{2\leq i\leq 2p}\left\{a_i, \frac{1}{a_i}\right\} \geq \min_{1\leq i\leq 2p-1}\left\{a_i, \frac{1}{a_i}\right\}, \\ a_{2p+1} &\leq \max\{h_{p-2}(a_2, \ldots, a_{2p}), h_{p-1}(a_2, \ldots, a_{2p})\} \leq \max_{2\leq i\leq 2p}\left\{a_i, \frac{1}{a_i}\right\} \leq \max_{1\leq i\leq 2p-1}\left\{a_i, \frac{1}{a_i}\right\}. \end{aligned} \tag{2.17}$$

Working inductively, we conclude that for $k = 2p, 2p+1, \ldots$,

$$a_k \geq \min\{h_{p-2}(a_{k-2p+1}, \ldots, a_{k-1}), h_{p-1}(a_{k-2p+1}, \ldots, a_{k-1})\} \geq \min_{1\leq i\leq 2p-1}\left\{a_i, \frac{1}{a_i}\right\}, \tag{2.18}$$

$$a_k \leq \max\{h_{p-2}(a_{k-2p+1}, \ldots, a_{k-1}), h_{p-1}(a_{k-2p+1}, \ldots, a_{k-1})\} \leq \max_{1\leq i\leq 2p-1}\left\{a_i, \frac{1}{a_i}\right\}. \tag{2.19}$$

□

Claim 2. If $a_{2p+1} = \max\limits_{1\le i\le 2p-1}\{a_i, 1/a_i\}$, then $a_1 = \cdots = a_{2p-1} = 1$.

Proof of Claim 2. By (2.19), we get

$$a_{2p+1} = \max\{h_{p-2}(a_2,\ldots,a_{2p}), h_{p-1}(a_2,\ldots,a_{2p})\} = \max_{1\le i\le 2p-1}\left\{a_i, \frac{1}{a_i}\right\}. \tag{2.20}$$

Here, we encounter two possible cases.

Case 1. $a_{2p+1} = h_{p-2}(a_2,\ldots,a_{2p}) = \max_{1\le i\le 2p-1}\{a_i, 1/a_i\}$. By Theorem 2.4, we get $a_2 = \cdots = a_{2p} = 1$ and, hence, $a_{2p+1} = 1$. Then $1 = a_{2p+1} = \max_{1\le i\le 2p-1}\{a_i, 1/a_i\} = \max\{a_1, 1/a_1\}$, implying $a_1 = 1$.

Case 2. $a_{2p+1} = h_{p-1}(a_2,\ldots,a_{2p}) = \max_{1\le i\le 2p-1}\{a_i, 1/a_i\}$. By Theorem 2.5, we get

$$a_2 = \cdots = a_{p+1} = \frac{1}{a_{p+2}} = \cdots = \frac{1}{a_{2p}}, \tag{2.21}$$

and consequently,

$$a_{2p+1} = h_{p-1}(a_2,\ldots,a_{2p}) = a_2. \tag{2.22}$$

Then,

$$\begin{aligned}\max_{1\le i\le 2p-1}\left\{a_i, \frac{1}{a_i}\right\} = a_{2p+1} = \frac{1}{a_{2p}} &\le \frac{1}{\min\{h_{p-2}(a_1,\ldots,a_{2p-1}), h_{p-1}(a_1,\ldots,a_{2p-1})\}}\\ &\le \max_{1\le i\le 2p-1}\left\{a_i, \frac{1}{a_i}\right\}.\end{aligned} \tag{2.23}$$

Hence, all equalities in this chain hold. In particular, we have

$$\min\{h_{p-2}(a_1,\ldots,a_{2p-1}), h_{p-1}(a_1,\ldots,a_{2p-1})\} = \min_{1\le i\le 2p-1}\left\{a_i, \frac{1}{a_i}\right\}. \tag{2.24}$$

If $h_{p-2}(a_1,\ldots,a_{2p-1}) = \min_{1\le i\le 2p-1}\{a_i, 1/a_i\}$, it follows from Theorem 2.4 that $a_1 = \cdots = a_{2p-1} = 1$. Now, assume that $h_{p-1}(a_1,\ldots,a_{2p-1}) = \min_{1\le i\le 2p-1}\{a_i, 1/a_i\}$. By Theorem 2.5, we get

$$a_1 = \cdots = a_p = \frac{1}{a_{p+1}} = \cdots = \frac{1}{a_{2p-1}}. \tag{2.25}$$

Equations (2.21) and (2.25) imply that $a_1 = \cdots = a_{2p-1} = 1$. Claim 2 is proven. □

By Claim 2 and working inductively, we get that if $a_k = \max_{1\le i\le 2p-1}\{a_i, 1/a_i\}$ for some $k \ge 2p+1$, then $a_1 = \cdots = a_{2p-1} = 1$.

Similarly, we can show that $a_1 = \cdots = a_{2p-1} = 1$ if $a_k = \min_{1\le i\le 2p-1}\{a_i, 1/a_i\}$ holds for some $k \ge 2p+1$.

As an application of Theorem 2.6, we have the following theorem.

Theorem 2.7. *Let $p \ge 3$, $p-2 \le w \le p-1$. The difference equation*

$$x_n = h_w(x_{n-2p+1},\ldots,x_{n-1}), \quad n = 1,2,\ldots, \tag{2.26}$$

with positive initial conditions admits the globally asymptotically stable equilibrium $c = 1$.

The proof of this theorem is similar to those in [11, 13], and hence is omitted.

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