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Research Article

A Boundary Value Problem for Hermitian Monogenic Functions

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We study the problem of finding a Hermitian monogenic function with a given jump on a given hypersurface in $\mathbb{R}^m$, $m = 2n$. Necessary and sufficient conditions for the solvability of this problem are obtained.

1. Introduction

Hermitian Clifford analysis deals with the simultaneous null solutions of the orthogonal Dirac operators $\partial_{\underline{x}}$ and its twisted counterpart $\partial_{\underline{x}|}$, introduced below. For a thorough treatment of this higher-dimensional function theory, we refer the reader to, for example, [1–5].

Let $(e_1, \ldots, e_{2n})$ be an orthonormal basis of the Euclidean space $\mathbb{R}^{2n}$. Consider the complex Clifford algebra $\mathbb{C}_{2n}$ constructed over $\mathbb{R}^{2n}$. The noncommutative multiplication in $\mathbb{C}_{2n}$ is governed by

$$\begin{gathered} e_j^2 = -1, \quad j = 1, \ldots, 2n, \\ e_j e_k + e_k e_j = 0, \quad 1 \leq j \neq k \leq 2n. \end{gathered} \tag{1.1}$$

A basis for $\mathbb{C}_{2n}$ is obtained by considering for a set $A = \{j_1, \ldots, j_k\} \subset \{1, \ldots, 2n\}$ the element $e_A = e_{j_1} \ldots e_{j_k}$, with $j_1 < \cdots < j_k$. For the empty set $\emptyset$, we put $e_\emptyset = 1$, the latter being the identity element.

Any Clifford number $a \in \mathbb{C}_{2n}$ may thus be written as

$$a = \sum_A a_A e_A, \quad a_A \in \mathbb{C}, \tag{1.2}$$

and its Hermitian conjugate $\overline{a}$ is defined by

$$\overline{a} = \sum_A \overline{a}_A \overline{e}_A, \quad \overline{e}_A = (-1)^{k(k+1)/2} e_A, \quad |A| = k. \tag{1.3}$$

The Euclidean space $\mathbb{R}^{2n}$ is embedded in the Clifford algebra $\mathbb{C}_{2n}$ by identifying $(x_1, \ldots, x_{2n})$ with the real Clifford vector $\underline{x}$ given by

$$\underline{x} = \sum_{j=1}^{n} \left(e_{2j-1} x_{2j-1} + e_{2j} x_{2j} \right). \tag{1.4}$$

The product of two vectors splits up into a scalar part and a so-called bivector part:

$$\underline{x}\underline{y} = -\langle \underline{x}, \underline{y} \rangle + \underline{x} \wedge \underline{y}, \tag{1.5}$$

where

$$\begin{aligned} \langle \underline{x}, \underline{y} \rangle &= \sum_{j=1}^{2n} x_j y_j, \\ \underline{x} \wedge \underline{y} &= \sum_{j=1}^{2n} \sum_{k=j+1}^{2n} e_j e_k \left(x_j y_k - x_k y_j \right). \end{aligned} \tag{1.6}$$

We also introduce for each real Clifford vector $\underline{x}$ its twisted counterpart

$$\underline{x}| = \sum_{j=1}^{n} \left(e_{2j-1} x_{2j} - e_{2j} x_{2j-1} \right). \tag{1.7}$$

Note that $\underline{x}^2 = -\langle \underline{x}, \underline{x} \rangle = -|\underline{x}|^2 = -|\underline{x}||^2 = \underline{x}|^2$. Also observe that the Clifford vectors $\underline{x}$ and $\underline{x}|$ are orthogonal with respect to the standard Euclidean scalar product, which implies that $\underline{x}\underline{x}| = -\underline{x}|\underline{x}$.

The Fischer dual of the vector $\underline{x}$ is the first-order differential operator

$$\partial_{\underline{x}} = \sum_{j=1}^{n} \left(e_{2j-1} \partial_{x_{2j-1}} + e_{2j} \partial_{x_{2j}} \right) \tag{1.8}$$

called Dirac operator. Null solutions of this operator are called monogenic functions, which may be regarded as a natural generalization to a higher-dimensional setting of the holomorphic functions of one complex variable (see [6, 7]). A function f continuously differentiable in an open set Ω of $\mathbb{R}^{2n}$ and taking value in $\mathbb{C}_{2n}$ is said to be (left) monogenic in Ω if and only if $\partial_{\underline{x}} f = 0$ in Ω. In a similar way, a notion of monogenicity can be associated to the Fischer dual of the vector $\underline{x}|$ given by

$$\partial_{\underline{x}|} = \sum_{j=1}^{n} \left(e_{2j-1} \partial_{x_{2j}} - e_{2j} \partial_{x_{2j-1}} \right). \tag{1.9}$$

We notice that the Dirac operators $\partial_{\underline{x}}$ and $\partial_{\underline{x}|}$ anticommute and factorize the Laplacian, that is, $-\partial_{\underline{x}}^2 = \Delta = -\partial_{\underline{x}|}^2$. Thus, monogenicity with respect to $\partial_{\underline{x}}$ (resp., $\partial_{\underline{x}|}$) can be regarded as a refinement of harmonicity.

Further, a continuously differentiable function f in an open set Ω of $\mathbb{R}^{2n}$ with values in $\mathbb{C}_{2n}$ is called a (left) Hermitian monogenic (or h-monogenic) function in Ω if and only if it satisfies in Ω the system

$$\partial_{\underline{x}} f = 0 = \partial_{\underline{x}|} f. \tag{1.10}$$

Throughout the paper Ω^+ will stand for an open-bounded set in $\mathbb{R}^{2n}$ with a boundary compact topological hypersurface Γ of finite $(2n-1)$-dimensional Hausdorff measure, and $\Omega^- = \mathbb{R}^{2n} \setminus \Omega^+$. We assume that both open sets $\Omega^\pm$ are connected. Finally, suppose that f belongs to the Hölder space $C^{0,\alpha}(\Gamma)$, $0 < \alpha < 1$.

The aim of this paper is to the study the following jump problem for h-monogenic functions. Under which conditions can we decompose a given f on Γ as

$$f = f^+ - f^-, \tag{1.11}$$

where $f^\pm \in C^{0,\alpha}(\Gamma)$ are extendable to h-monogenic functions $F^\pm$ in $\Omega^\pm$ with $F^-(\infty) = 0$?

First, it should be noticed that if this jump problem has a solution, then it is unique. This assertion can be easily proved using the Painlevé and Liouville theorems in the Clifford analysis setting (see [6, 8]).

This work is motivated by the results obtained in [9, 10] where a similar problem was studied for two-sided monogenic functions. For the case of harmonic vector fields, we refer the reader to [11].

In order to solve problem (1.11), we propose two different approaches. The first one uses an integral criterion for h-monogenicity (Section 2); and for the second approach, we establish a conservation law for h-monogenic functions (Section 3).

2. An integral criterion for h-monogenicity

Let us denote by $\mathscr{H}^{2n-1}$ the $(2n-1)$-dimensional Hausdorff measure (see [12–14]). In this section, we require Γ to be an Ahlfors-David regular hypersurface (see [15]), that is, there exists $c > 0$ such that for all $\underline{x} \in \Gamma$ and all $0 < r \leq \operatorname{diam} \Gamma$,

$$c^{-1} r^{2n-1} \leq \mathscr{H}^{2n-1}\left(\Gamma \cap \left\{|\underline{y} - \underline{x}| \leq r\right\}\right) \leq c r^{2n-1}. \tag{2.1}$$

The fundamental solutions of the Dirac operators $\partial_{\underline{x}}$ and $\partial_{\underline{x}|}$ introduced in the previous section are, respectively,

$$E(\underline{x}) = -\frac{1}{\sigma_{2n}} \frac{\underline{x}}{|\underline{x}|^{2n}}, \qquad E|(\underline{x}) = -\frac{1}{\sigma_{2n}} \frac{\underline{x}|}{|\underline{x}|^{2n}}, \tag{2.2}$$

where σ_{2n} is the surface area of the unit sphere S^{2n-1} in $\mathbb{R}^{2n}$.

Let us consider the following Cauchy-type integrals $\mathbf{C}_\Gamma f$, $\mathbf{C}_\Gamma | f$, and their singular versions $\mathbf{S}_\Gamma f$, $\mathbf{S}_\Gamma | f$, defined as

$$\begin{aligned}
(\mathbf{C}_\Gamma f)(\underline{x}) &= \int_\Gamma E(\underline{y} - \underline{x}) \underline{\nu}(\underline{y}) f(\underline{y}) d\mathscr{H}^{2n-1}(\underline{y}), \\
(\mathbf{S}_\Gamma f)(\underline{z}) &= 2 \lim_{\epsilon \to 0+} \int_{\Gamma \setminus \{|\underline{y} - \underline{z}| \leq \epsilon\}} E(\underline{y} - \underline{z}) \underline{\nu}(\underline{y}) \left(f(\underline{y}) - f(\underline{z})\right) d\mathscr{H}^{2n-1}(\underline{y}) + f(\underline{z}), \\
(\mathbf{C}_\Gamma | f)(\underline{x}) &= \int_\Gamma E\big|(\underline{y} - \underline{x}) \underline{\nu}\big|(\underline{y}) f(\underline{y}) d\mathscr{H}^{2n-1}(\underline{y}), \\
(\mathbf{S}_\Gamma | f)(\underline{z}) &= 2 \lim_{\epsilon \to 0+} \int_{\Gamma \setminus \{|\underline{y} - \underline{z}| \leq \epsilon\}} E\big|(\underline{y} - \underline{z}) \underline{\nu}\big|(\underline{y}) \left(f(\underline{y}) - f(\underline{z})\right) d\mathscr{H}^{2n-1}(\underline{y}) + f(\underline{z}),
\end{aligned} \tag{2.3}$$

for $\underline{x} \in \mathbb{R}^{2n} \setminus \Gamma$ and $\underline{z} \in \Gamma$.

Here and subsequently, $\underline{\nu}(\underline{y}) = \sum_{j=1}^{n}(e_{2j-1}\nu_{2j-1}(\underline{y}) + e_{2j}\nu_{2j}(\underline{y}))$ stands for the unit normal vector on Γ at the point $\underline{y}$ introduced by Federer (see [13]).

Note that $\mathbf{C}_\Gamma f$ (resp., $\mathbf{C}_\Gamma|f$) is monogenic in $\mathbb{R}^{2n} \setminus \Gamma$ with respect to $\partial_{\underline{x}}$ (resp., $\partial_{\underline{x}|}$) and that $\mathbf{C}_\Gamma f(\infty) = \mathbf{C}_\Gamma|f(\infty) = 0$.

Let us now formulate some important properties of these integral operators. For their proofs, we refer the reader to [16, 17].

(a) $\mathbf{S}_\Gamma f$, $\mathbf{S}_\Gamma|f \in C^{0,\alpha}(\Gamma)$.

(b) Sokhotski-Plemelj formulae: for $\underline{z} \in \Gamma$,

$$\begin{aligned}(\mathbf{C}_\Gamma^{\pm} f)(\underline{z}) &= \lim_{\Omega^\pm \ni \underline{x} \to \underline{z}} (\mathbf{C}_\Gamma f)(\underline{x}) = \frac{1}{2}((\mathbf{S}_\Gamma f)(\underline{z}) \pm f(\underline{z})),\\ (\mathbf{C}_\Gamma|^{\pm} f)(\underline{z}) &= \lim_{\Omega^\pm \ni \underline{x} \to \underline{z}} (\mathbf{C}_\Gamma|f)(\underline{x}) = \frac{1}{2}((\mathbf{S}_\Gamma|f)(\underline{z}) \pm f(\underline{z})).\end{aligned} \tag{2.4}$$

Theorem 2.1 (integral criterion). *The function f has an h-monogenic extension $F^\pm$ in $\Omega^\pm$, $F^-(\infty) = 0$, if and only if $\mathbf{S}_\Gamma f = \pm f = \mathbf{S}_\Gamma|f$.*

Proof. Suppose that f has an h-monogenic extension F^+ in Ω^+. By Cauchy's integral formula for monogenic functions (see [6]), we have

$$(\mathbf{C}_\Gamma f)(\underline{x}) = F^+(\underline{x}) = (\mathbf{C}_\Gamma|f)(\underline{x}), \quad \underline{x} \in \Omega^+. \tag{2.5}$$

Property (b) now implies

$$\mathbf{S}_\Gamma f = f = \mathbf{S}_\Gamma|f. \tag{2.6}$$

Conversely, assume that $\mathbf{S}_\Gamma f = f = \mathbf{S}_\Gamma|f$. From (2.6) and using again property (b), we obtain

$$\mathbf{C}_\Gamma^{+} f = f = \mathbf{C}_\Gamma|^{+} f. \tag{2.7}$$

Note that $\mathbf{C}_\Gamma f - \mathbf{C}_\Gamma|f$ is harmonic in Ω^+ and $\mathbf{C}_\Gamma^{+} f - \mathbf{C}_\Gamma|^{+} f = 0$. The maximum and the minimum principle for harmonic functions now yields $\mathbf{C}_\Gamma f = \mathbf{C}_\Gamma|f$ in Ω^+, hence that $\mathbf{C}_\Gamma f$ is h-monogenic in Ω^+. Therefore by putting

$$F^+(\underline{x}) = \begin{cases}(\mathbf{C}_\Gamma f)(\underline{x}), & \underline{x} \in \Omega^+,\\ f(\underline{x}), & \underline{x} \in \Gamma,\end{cases} \tag{2.8}$$

we obtain an h-monogenic extension of f in Ω^+. The case Ω^- is proved similarly. □

We are now in the position to give a first solution to (1.11). We first claim that if f can be decomposed as in (1.11), then $\mathbf{S}_\Gamma f = \mathbf{S}_\Gamma|f$. Indeed, Theorem 2.1 now leads to

$$\mathbf{S}_\Gamma f = \mathbf{S}_\Gamma f^+ - \mathbf{S}_\Gamma f^- = \mathbf{S}_\Gamma\big|f^+ - \mathbf{S}_\Gamma\big|f^- = \mathbf{S}_\Gamma\big|f. \tag{2.9}$$

On the other hand, if $\mathbf{S}_\Gamma f = \mathbf{S}_\Gamma|f$, then an analysis similar to that in the proof of Theorem 2.1 shows that $\mathbf{C}_\Gamma f = \mathbf{C}_\Gamma|f$, which implies that $\mathbf{C}_\Gamma f$ is h-monogenic in $\mathbb{R}^{2n} \setminus \Gamma$. Finally, by (a) and (b), we conclude that $f^\pm = \mathbf{C}_\Gamma^{\pm} f = \mathbf{C}_\Gamma|^{\pm} f$ is a solution of the jump problem (1.11).

Summarizing, we have the following.

Theorem 2.2. *The following statements are equivalent:*

(i) f *can be decomposed as in* (1.11)*;*

(ii) $\mathbf{S}_\Gamma f = \mathbf{S}_\Gamma | f$*;*

(iii) $\mathbf{C}_\Gamma f = \mathbf{C}_\Gamma | f$*;*

(iv) $\mathbf{C}_\Gamma f$ *is h-monogenic in* $\mathbb{R}^{2n} \setminus \Gamma$.

Moreover, if the jump problem (1.11) *is solvable, then its unique solution is given by*

$$f^{\pm} = \mathbf{C}_\Gamma^{\pm} f = \frac{1}{2}\left(\mathbf{S}_\Gamma f \pm f\right) = \mathbf{C}_\Gamma|^{\pm} f = \frac{1}{2}\left(\mathbf{S}_\Gamma | f \pm f\right). \tag{2.10}$$

3. A conservation law for h-monogenic functions

In the remainder of this paper, we assume Γ to be a C^1-smooth hypersurface. Then for $\underline{x}$ sufficiently close to Γ, we may assume that the orthogonal projection of $\underline{x}$ onto Γ is unique and it is denoted by $\underline{x}_\perp$. Let us denote by $\underline{\nu} = \sum_{j=1}^{n}(e_{2j-1}\nu_{2j-1} + e_{2j}\nu_{2j})$ the unit normal vector on Γ at the point $\underline{x}_\perp$.

In a neighborhood of Γ, we have the decomposition of $\partial_{\underline{x}}$ in the normal and the tangential parts (see [18])

$$\partial_{\underline{x}} = -\underline{\nu}\left(\underline{\nu}\partial_{\underline{x}}\right) = \underline{\nu}\partial_\nu + \partial_{\|\underline{x}}, \tag{3.1}$$

where

$$\partial_\nu = \left\langle \underline{\nu}, \partial_{\underline{x}} \right\rangle, \qquad \partial_{\|\underline{x}} = -\underline{\nu}\left(\underline{\nu} \wedge \partial_{\underline{x}}\right). \tag{3.2}$$

Similarly,

$$\partial_{\underline{x}|} = -\underline{\nu}|\left(\underline{\nu}|\partial_{\underline{x}|}\right) = \underline{\nu}|\partial_\nu + \partial_{\|\underline{x}|}, \tag{3.3}$$

with

$$\partial_{\|\underline{x}|} = -\underline{\nu}|\left(\underline{\nu}| \wedge \partial_{\underline{x}|}\right). \tag{3.4}$$

The restrictions of the operators $\partial_{\|\underline{x}}$ and $\partial_{\|\underline{x}|}$ to Γ will be denoted by $\partial_{\underline{\omega}}$ and $\partial_{\underline{\omega}|}$, respectively.

Let us suppose at the outset that $F \in C^1(\overline{\Omega^+})$ is a monogenic function in Ω^+ with respect to $\partial_{\underline{x}}$ and set $g = F|_\Gamma$. If F is moreover h-monogenic in Ω^+, then from (3.1) and (3.3), we obtain that in a neighbourhood of Γ intersected with Ω^+

$$\partial_\nu F - \underline{\nu}\partial_{\|\underline{x}} F = 0, \qquad \partial_\nu F - \underline{\nu}|\partial_{\|\underline{x}|} F = 0. \tag{3.5}$$

In this way, $\underline{\nu}\,\partial_{\|\underline{x}} F = \underline{\nu}|\partial_{\|\underline{x}|} F$ in a neighbourhood of Γ intersected with Ω^+. By continuity, we get on Γ the relation

$$\underline{\nu}|\underline{\nu}\,\partial_{\underline{\omega}} g + \partial_{\underline{\omega}|} g = 0. \tag{3.6}$$

On the other hand, if g satisfies (3.6), then for $G = \partial_{\underline{x}|}F$, we have

$$G = \underline{\nu}|\partial_\nu F + \partial_{\|\underline{x}|}F, \qquad 0 = \underline{\nu}\,\partial_\nu F + \partial_{\|\underline{x}}F. \tag{3.7}$$

Therefore in a neighbourhood of Γ intersected with Ω^+, we obtain

$$G = \underline{\nu}|\underline{\nu}\,\partial_{\|\underline{x}}F + \partial_{\|\underline{x}|}F. \tag{3.8}$$

It follows immediately that $G|_\Gamma = \nu|\underline{\nu}\,\partial_{\underline{\omega}}g + \partial_{\underline{\omega}|}g = 0$. As G is h-monogenic in Ω^+ and hence harmonic, we conclude that $\partial_{\underline{x}|}F = G = 0$ in Ω^+.

Note that this analysis may be also applied to monogenic functions in Ω^- with respect to $\partial_{\underline{x}}$ vanishing at infinity.

We have thus proved the following.

Theorem 3.1 (conservation law). *Let $F^\pm \in C^1(\overline{\Omega^\pm})$ be a monogenic function in $\Omega^\pm$ with respect to $\partial_{\underline{x}}$, $F^-(\infty) = 0$. Then, $F^\pm$ is an h-monogenic function in $\Omega^\pm$ if and only if $g = F^\pm|_\Gamma$ satisfies* (3.6).

Let us return to the jump problem (1.11). If f can be decomposed as in (1.11), then Theorem 3.1 now gives

$$\underline{\nu}|\underline{\nu}\,\partial_{\underline{\omega}}f + \partial_{\underline{\omega}|}f = \left(\underline{\nu}|\underline{\nu}\,\partial_{\underline{\omega}}f^+ + \partial_{\underline{\omega}|}f^+\right) - \left(\underline{\nu}|\underline{\nu}\,\partial_{\underline{\omega}}f^- + \partial_{\underline{\omega}|}f^-\right) = 0. \tag{3.9}$$

Conversely, suppose that $\underline{\nu}|\underline{\nu}\,\partial_{\underline{\omega}}f + \partial_{\underline{\omega}|}f = 0$. Define $f^\pm = \mathbf{C}^\pm_\Gamma f$. We will prove that $f^\pm$ is a solution of (1.11). To do this, take $G = \partial_{\underline{x}|}\mathbf{C}_\Gamma f$. It follows that

$$G = \underline{\nu}|\underline{\nu}\,\partial_{\|\underline{x}}\mathbf{C}_\Gamma f + \partial_{\|\underline{x}|}\mathbf{C}_\Gamma f. \tag{3.10}$$

Consequently, the limit values $G^\pm$ of G taken from $\Omega^\pm$ are given by

$$G^\pm = \underline{\nu}|\underline{\nu}\,\partial_{\underline{\omega}}\mathbf{C}^\pm_\Gamma f + \partial_{\underline{\omega}|}\mathbf{C}^\pm_\Gamma f. \tag{3.11}$$

From (b) we see that $G^+ - G^- = \underline{\nu}|\underline{\nu}\,\partial_{\underline{\omega}}f + \partial_{\underline{\omega}|}f = 0$. As the function G is h-monogenic in $\mathbb{R}^{2n} \setminus \Gamma$ and vanishes at infinity, we have $G \equiv 0$ in $\mathbb{R}^{2n} \setminus \Gamma$, the last equality being a consequence of the Painlevé and Liouville theorems.

We thus arrive to another characterization for the solvability of the jump problem (1.11).

Theorem 3.2. *The jump problem* (1.11) *is solvable if and only if*

$$\underline{\nu}|\underline{\nu}\,\partial_{\underline{\omega}}f + \partial_{\underline{\omega}|}f = 0. \tag{3.12}$$

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