NEW APPROXIMATE SOLUTIONS PER UNIT OF TIME FOR PERIODICALLY CHECKED SYSTEMS WITH DIFFERENT LIFETIME DISTRIBUTIONS

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Systems with different lifetime distributions, associated with increasing, decreasing, constant, and bathtub-shaped hazard rates, are examined in this paper. It is assumed that a failure is only detected if systems are inspected. New approximate solutions for the inspection period and for the expected duration of hidden faults are presented, on the basis of the assumption that only periodic and perfect inspections are carried out. By minimizing total expected cost per unit of time, on the basis of numerical results and a range of comparisons, the conclusion is drawn that these new approximate solutions are extremely useful and simple to put into practice.

1. Introduction

Let us consider a system, which may be of any type or nature, whose lifetime is a continuous random variable T with density function $f(t)$ and expected value $E(T)$. If $F(t)$ is the respective distribution function, then the reliability function $R(t)$ and the hazard rate $h(t)$ are given by

$$R(t) = 1 - F(t); \qquad h(t) = \frac{f(t)}{R(t)}. \tag{1.1}$$

Let us assume that a perfect or imperfect system state is only detected if the system is inspected and that inspections are perfect and periodic.

We will regard a cycle as the interval of time between the initial instant and the instant at which a failure is detected and corrected by means of an inspection. Let us assume that system state is not changed by inspections and that their duration is nil. A failure having been detected and corrected, we will assume that the system is as good as new; a renewal

process thus takes place and a new cycle begins. In this paper, we will analyze the system operation per unit of time over the duration of a cycle.

The proposed solutions that will be presented in this paper are applicable in a wide variety of fields. Engineering, medicine, and maintenance scenarios would benefit from these procedures where the goal is the minimization of expected costs in systems whose lifetime is a random variable and where failures may occur. For example, they could be applied in the detection of

> (a) failures in nuclear systems;
> (b) enemy missiles or airplanes;
> (c) forest fires and fires in buildings;
> (d) global or species-specific diseases affecting people, birds, and other animals; current developments and recent experience highlight the importance of finding such solutions.

If N is the number of system inspections per cycle (including the inspection at whose instant a failure is detected), and if D represents the detection time of a hidden fault (between the instant of system failure and the instant of its detection during the next inspection), which are both random variables, while P is the inspection period (with inspection times over P, $2P$, $3P$, …), then their expected values $E(\cdot)$ are given by

$$E(N) = \sum_{k=0}^{\infty}(k+1)\int_{kP}^{(k+1)P} f(t)dt = \sum_{k=0}^{\infty} R(kP), \qquad E(D) = P \cdot E(N) - E(T). \qquad (1.2)$$

If C_1 is the cost of each inspection and C_2 is the cost per unit of time of imperfect operation before system failure is detected, then the total expected cost per cycle $E(C) = C_1 E(N) + C_2 E(D)$ is minimized for a value of P, which is a solution of the following equation (see Rodrigues Dias [7]):

$$\sum_{k=0}^{\infty} R(kP) - (r+P)\sum_{k=0}^{\infty} kf(kP) = 0, \quad r = \frac{C_1}{C_2}. \qquad (1.3)$$

It should be noted that there may be one, two, or more solutions, which do not depend on the individual values of costs C_1 and C_2 (interestingly, they depend on their quotient r).

This problem was initially presented and examined by Barlow et al. [3].

However, it has been shown that it is extremely difficult or perhaps impossible to analytically obtain a solution for (1.3), even when T has an exponential distribution. Therefore, numerical approaches are required.

In (1.2), a simple and useful geometric interpretation of $E(D)$ is presented. In fact, $E(T) = \int_0^{\infty} R(t)dt$ is the area below the reliability function curve $R(t)$ and above the x axis for values between 0 and ∞; meanwhile, $P \cdot E(N) = \sum_{k=0}^{+\infty} P \cdot R(kP)$ is the sum of the area of the rectangles whose base is P centered on kP ($k = 0, 1, 2, …$) and whose height is $R(kP)$. Therefore, $E(D)$ is the difference between the latter area and the former area, with regard to $E(T)$. The straightforward conclusion is that $E(D)$ is approximately equal to $P/2$; however, it may be either larger or smaller than half the inspection period P. This

depends on the shape of the reliability function $R(t)$. There is no general expression which approximates to $E(D)$.

Thus, starting with the simplest approach $E(D) \cong P/2$ (see Schneeweiss [15]), Nakagawa and Yasui [6] obtained the following approximate solution for the inspection period P_{C1} per cycle (with $E(N) \cong E(T)/P + 1/2$):

$$P_{C1} = \sqrt{2rE(T)}. \tag{1.4}$$

In a later work (see Rodrigues Dias [8]), in which T has an exponential distribution, based on the analysis of results previously obtained (see Rodrigues Dias [7]), a new approximate solution for the inspection period was obtained by using the least-square method which minimizes the total expected cost per cycle:

$$P_{C2} = \frac{\sqrt{2rE(T)}}{1 + 0.234\sqrt{r'}} = \frac{P_{C1}}{G}, \quad G = 1 + 0.234\sqrt{r'}; \; r' = \frac{r}{E(T)}. \tag{1.5}$$

This solution is demonstrated to be nearly optimal because its relative error is approximately equal to zero in terms of costs. It can be regarded as a generalization of the approach developed by Nakagawa and Yasui [6], where G is the generalization factor.

The practical usefulness of simple approaches should be stressed, as can be seen in a quality control context of Saniga and Shirland in [14].

Problems of the same type can be found in Badía et al. [2], Dieulle [4], Rodrigues Dias [9], Rodrigues Dias and Infante [12], Sheu et al. [16], and Vaurio [17], for example. In Rodrigues Dias [10, 11], two new approaches dealing with different sampling intervals in quality control are proposed.

2. A new approximate solution for the inspection period

Let us now discuss whether it is possible to obtain solutions which in some way correspond to those previously presented in (1.4) and (1.5), with a view to minimizing expected total cost per unit of time of the cycle. In fact, the duration of each cycle is a random variable, resulting from system lifetime T plus the duration of a hidden fault D.

Hence, if C_U is the total cost per unit of time and $E(C_U)$ is its expected value, and there is a renewal process (see Ross [13]), we have

$$E(C_U) = \frac{C_1 E(N) + C_2 E(D)}{E(T) + E(D)}. \tag{2.1}$$

What must be determined is the value of period P that minimizes the function given by (2.1). With regard to the minimization of expected total cost per cycle, in general, an analytical solution cannot be obtained. Therefore, it is essential that approximate solutions be obtained and that an evaluation be carried out, for different situations, as to whether they are satisfactory from a practical point of view.

Let us suppose that $E(D) \cong P/2$. If $E(N) \cong E(T)/P + 1/2$, the following expression can easily be obtained:

$$E(C_U) \cong \frac{2C_1E(T) + C_1P + C_2P^2}{2PE(T) + P^2}. \tag{2.2}$$

By calculating the respective derivative associated with P and making it equal to zero, the following second-order equation can be obtained:

$$[2C_2E(T) - C_1] \cdot P^2 - 4C_1E(T) \cdot P - 4C_1E(T)^2 = 0. \tag{2.3}$$

As one of the two solutions will always be negative, the other one can be expressed as follows:

$$P = \frac{[2rE(T) + 2E(T) \cdot \sqrt{2rE(T)}\,]}{[2E(T) - r]}. \tag{2.4}$$

By means of a suitable convenient algebraic transformation, assuming for instance an auxiliary variable z, such that $z^2 = 2rE(T)$, the simple solution P_{U1} can be obtained:

$$P_{U1} = \frac{2rE(T)}{\sqrt{2rE(T)} - r} = \frac{P_{C1}^2}{P_{C1} - r}. \tag{2.5}$$

Thus, P_{U1} represents the first approximate solution presented in this paper for period P that minimizes expected total cost per unit of time.

Interestingly, it is possible to express P_{U1} as a function of P_{C1} and r. Even more interestingly, when r tends towards zero (which may mean increasingly lower costs for each inspection), the two expressions P_{U1} and P_{C1} get closer together. In a sense, it can be said that the approximate solution we have obtained, given by (2.5), is a generalization of the approach given in (1.4).

As a simple example, let us consider a system with an average lifetime $E(T) = 1000$ hours, the numeric value of each inspection cost C_1 being equal to 1 and the numeric value of cost per hour of imperfect operation C_2 being equal to 20. Thus, very simply, we have $r = 0.05$ and from (2.5), we can calculate that $P_{U1} \cong 10.05$ hours. It should be noted that no assumptions are made here about the type of distribution of T.

This new approximate solution will be analyzed in the following section.

3. Results obtained and analysis of results

We will present some numerical results and analyze, in brief, whether the new approximate solution for the inspection period P_{U1} can be regarded as satisfactory. To this end, we will consider different situations. Firstly, let us attribute a different degree of significance to the cost of each inspection C_1, comparing each with the cost per unit of time of imperfect operation C_2. In fact, we will use different values of r', given in (1.5), which is a dimensionless parameter, as can be seen in the following tables. This means taking $E(T)$ as the unit of time. Secondly, we will consider the case in which the system has different failure rates $h(t)$, for three different lifetime distributions: (a) the Weibull, (b) the log-normal, and (c) the Hjorth [5] distributions.

(a) The Weibull distribution is a model very often used in this kind of reliability context. Interestingly, this distribution, whose shape parameter is equal to 2.5, is used to model the incubation period of the HIV1virus (see Amaral et al. [1]).

In the Weibull distribution, α being a scale parameter and β a form parameter, the reliability function is given by

$$R(t) = \exp\left(\frac{-t}{\alpha}\right)^{\beta}, \quad t \geq 0. \tag{3.1}$$

In this paper, let us assume that form parameter β has the following values: $0.7, 1, 2, 3, 4$, and 5. System hazard rate $h(t)$ is decreasing where it is less than 1, and where it is greater than 1, $h(t)$ is increasing. When $\beta = 1$, we have an exponential distribution corresponding to a constant hazard rate $h(t) = 1/\alpha$. Finally, when the form parameter is close to 3, there is a degree of approach to normal distribution.

(b) In a system in which the lifetime distribution is log-normal, we have

$$R(t) = 1 - \Phi\left(\frac{\ln(t) - \mu}{\sigma}\right), \quad t \geq 0, \tag{3.2}$$

where $\Phi(\cdot)$ is the standard normal distribution function and μ and σ are two parameters. Interestingly, with the log-normal distribution, $h(t)$ increases initially but then decreases.

(c) The Hjorth [5] lifetime distribution, depending on 3 parameters δ, γ, and θ, has a reliability function given by

$$R(t) = \exp\left(-\frac{\delta t^2}{2}\right) \cdot (1 + \gamma t)^{-\theta/\gamma}, \quad t \geq 0. \tag{3.3}$$

This is significant because $h(t)$ can present a range of very different shapes. In particular, if $\theta = \gamma = 1$, and if $\delta = 0.01$, then $h(t)$ is bathtub-shaped.

Thus, the above three distributions allow us to consider all possible shapes of system failure rate $h(t)$.

Now, let us begin by comparing the approach P_{U1} given by (2.5) with the exact solution P_U obtained minimizing $E(C_U)$ given by (2.1), $E(N)$ and $E(D)$ being obtained from (1.2). Usually, when $E(N)$ increases, $E(D)$ decreases. This "exact" solution is calculated using numerical methods. Thus, we consider the quotient Q_1, given by the following expression, which gives the deviation, in relative terms and in percentage terms, between the two solutions for the inspection period:

$$Q_1 = \frac{P_{U1} - P_U}{P_U} \cdot 100\%. \tag{3.4}$$

Secondly, let us analyze the sensitivity of $E(C_U)$ as a function of P, in terms of evaluating the validity of the approximate solution P_{U1} in practical terms. To this end, let us consider Q_2, which gives the relative error, as a percentage, of the expected total cost associated with the solution P_{U1}, when compared with minimum cost, for the exact

Table 3.1. Values of relative errors Q_1, Q_2, and Q_3, in percentages, for different values of β of the Weibull distribution and for four values of r'.

β	$r' = 0.0125$			$r' = 0.0250$			$r' = 0.0500$			$r' = 0.1000$		
	Q_1	Q_2	Q_3	Q_1	Q_2	Q_3	Q_1	Q_2	Q_3	Q_1	Q_2	Q_3
0.7	7.3	0.2	8.4	9.0	0.3	10.5	11.2	0.5	13.1	13.6	0.6	16.4
1	2.7	0.0	2.7	3.9	0.1	3.9	5.7	0.1	5.6	8.3	0.2	8.1
2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0	0.0	−0.1	0.0	−0.1	−0.6	0.0	−0.2
4	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0	−7.2	0.2	−1.0
5	0.0	0.0	0.0	0.0	0.0	0.0	−3.2	0.0	0.0	−8.0	1.1	−5.0

solution P_U:

$$Q_2 = \frac{E(C_{U1}) - E(C_U)}{E(C_U)} \cdot 100\%. \tag{3.5}$$

Thirdly, as P_{U1} was obtained by considering $E(D) \cong P/2$, we are keen to evaluate this assumption. Thus, with $E(D_U)$ as the expected detection time of a system failure and P_U the optimum inspection period, we define Q_3 as

$$Q_3 = \frac{E(D_U) - P_U/2}{E(D_U)} \cdot 100\%. \tag{3.6}$$

Results obtained for Q_1, Q_2, and Q_3 are shown in Table 3.1, for the Weibull distribution, for four values of r'.

It can be shown that for values of β which approximate to 2, 3, and 4, if r' is not great (which means a very low expected number of inspections per cycle, which is unusual in practical applications), the relative errors of P_{U1} are practically nil in absolute values: they do not exceed 0.1% ($r' \leq 0.05$). The same can be shown for greater values of β, since r' values are not great. Meanwhile, when the system failure rate is constant ($\beta = 1$), it can be shown that values of Q_1 are significant, increasing whenever r' increases.

Now, let us analyze what happens with Q_2. It can be easily shown that the relative errors of expected total costs per unit of time are almost always nil (to one decimal place). Interestingly, even when the value of Q_1 is 13.6%, the corresponding value of Q_2 does not exceed 0.6%. This means that $E(C_U)$ shows a low degree of sensitivity to variations in period P, which clearly increases the significance of approximate solutions, especially as they are simple and can be rapidly obtained.

To conclude our analysis of the P_{U1} approach, let us see what happens with Q_3. It can typically be shown that the better the approximation $E(D) \approx P/2$ is, the better the approach given by (2.5) is. At the same time, it can be shown that $E(D)$ may be larger or smaller than $P/2$, depending on the distribution used. Meanwhile, in the case of exponential distributions, it is always greater, increasing as the value of r' increases.

Now, in Table 3.2, with regard to the log-normal and Hjorth distributions, relative errors Q_1, Q_2, and Q_3 are shown only for 2 values of r', since these are sufficient for arriving at a conclusion.

Table 3.2. Values of the relative errors Q_1, Q_2, and Q_3, in percentages, for the log-normal and the Hjorth distributions and for $r' = 0.0125$ and $r' = 0.0500$.

Distribution	$r' = 0.0125$			$r' = 0.0500$		
	Q_1	Q_2	Q_3	Q_1	Q_2	Q_3
Log-normal	-0.3	0.0	-0.3	2.4	0.0	0.4
Hjorth	-2.2	0.2	6.7	-6.1	0.5	11.8

Among other possible conclusions, we would point to the following.

(a) Depending on values of r' and the system lifetime distribution, values of the P_{U1} solution may be smaller or larger than those of the exact solution P_U; the same conclusion may be drawn with regard to $E(D_U)$, if it is compared with $P_U/2$.

(b) The type of lifetime distribution may be of great significance, as can be seen for $r' = 0.05$.

(c) Finally, values of Q_2 are small, and may be extremely small; this generally means that approximate solutions for the inspection period are always significant for all shapes of the system hazard rate $h(t)$.

4. Systems with a constant hazard rate: some new approximate solutions

In a similar manner to what was done above using (1.5), when the system lifetime is an exponential distribution with a constant hazard rate $h(t) = 1/\alpha$, let us introduce a second new approach P_{U2} given by

$$P_{U2} = \frac{P_{U1}}{1 + 0.234\sqrt{r'}} = \frac{P_{U1}}{G}. \tag{4.1}$$

It should be possible to search for another generalization factor which is slightly different from G. However, it seems best to keep this one as it has the advantage of being the same as in the case of the solution given by (1.5).

In order to determine its degree of accuracy, let us calculate relative errors Q_4:

$$Q_4 = \frac{P_{U2} - P_U}{P_U} \cdot 100\%. \tag{4.2}$$

Some values for Q_4 are presented in Table 4.1.

As can be seen from the table, with Q_4 approximately equal to zero, P_{U2} is a nearly optimal solution for P_U.

Having concluded that both P_{C2} and P_{U2} are nearly optimal solutions when the system has a constant hazard rate, we will now move on to consider the following questions. Are values of P_{C2} and P_{U2} very different? In other words, is there a difference in terms of minimizing total expected cost of the cycle and unit of time? If there is a difference, what is its magnitude? In order to provide a quick answer to these questions, let us calculate

Table 4.1.   Values of relative errors Q_4, Q_5, Q_6, Q_7, and Q_8, in percentages, for different values of r' and $\beta = 1$.

	$r' = 0.0125$	$r' = 0.0250$	$r' = 0.0500$	$r' = 0.1000$
Q_4	0.1	−0.2	−0.4	−0.8
Q_5	8.6	12.6	18.8	28.8
Q_6	−0.0861	−0.2078	−0.3879	−0.7562
Q_7	−0.0045	−0.0152	0.0547	0.0757
Q_8	−0.0104	−0.0271	0.0309	0.0278

the relative differences between P_{U2} and P_{C2}, using Q_5 given by

$$Q_5 = \frac{P_{U2} - P_{C2}}{P_{C2}} \cdot 100\% = \frac{1}{\sqrt{2/r'} - 1} \cdot 100\%. \tag{4.3}$$

Some calculated values are presented in Table 4.1.

In this case, as in other situations, it is obvious that Q_5 values show a considerable degree of difference, as do values for optimal periods P_C and P_U, with $P_U > P_C$. Thus, a key conclusion can now be drawn: it is essential to specify how optimal inspection times are calculated.

Finally, let us look at values of Q_1 and Q_3 shown in Table 3.1 when $\beta = 1$. It can easily be shown, especially if r' is small, that $Q_1 \cong Q_3$. Hence, on the basis of (3.4) and (3.6), we have

$$\frac{P_{U1} - P_U}{P_U} \cong \frac{E(D_U) - P_U/2}{E(D_U)}. \tag{4.4}$$

As values of Q_4 are approximately equal to zero, we can consider $P_U \cong P_{U2}$ and on the basis of (4.1), $P_{U1} \cong G \cdot P_U$. Using (4.4), and carrying out some analysis, the approximate solutions $E(D_U)_1$ and $E(D_U)_2$ can be obtained for the expected duration of hidden faults $E(D_U)$:

$$E(D_U)_1 = \frac{P_U}{2(1 - 0.234\sqrt{r'})}, \tag{4.5}$$

$$E(D_U)_2 = \frac{P_{U1}}{2G(1 - 0.234\sqrt{r'})}. \tag{4.6}$$

Similarly, considering that in (4.6), we have

$$G \cdot (1 - 0.234r'^{1/2}) = (1 + 0.234r'^{1/2}) \cdot (1 - 0.234r'^{1/2}) \cong (1 - 0.05r'), \tag{4.7}$$

then a third simple approximation for $E(D_U)_3$ can be derived:

$$E(D_U)_3 = \frac{P_{U1}}{2(1 - 0.05r')}. \tag{4.8}$$

In order to evaluate how satisfactory these solutions are, let us consider Q_6, Q_7, and Q_8 given by

$$Q_6 = \frac{E(D_U)_1 - E(D_U)}{E(D_U)} \cdot 100\%,$$

$$Q_7 = \frac{E(D_U)_2 - E(D_U)}{E(D_U)} \cdot 100\%, \tag{4.9}$$

$$Q_8 = \frac{E(D_U)_3 - E(D_U)}{E(D_U)} \cdot 100\%.$$

Some values are shown in Table 4.1.

From Table 3.1, it can be concluded that $E(D_U) > P_U/2$. Other conclusions can easily be drawn. In all cases, approximate solutions $E(D_U)_1$, $E(D_U)_2$, and $E(D_U)_3$ can be considered as being nearly optimal. $E(D_U)_2$ and $E(D_U)_3$ are quite similar approaches in terms of relative error: one of them is better for smaller values of r' and the other is better for larger values of r'. Finally, $E(D_U)_2$ contains the constant G, which is a possible advantage, and is consistent with other results presented here. Meanwhile, $E(D_U)_3$ is certainly the simplest approach.

5. Conclusions

In order to minimize the expected total cost of the operation of a system whose operational state can only be determined by means of inspections, it must be inspected either periodically or not. In this paper we consider the case of periodic inspections with two costs: one pertaining to the cost of each inspection and the other pertaining to the cost per unit of time of imperfect operation, following a system failure, before failure is detected and corrected. System lifetime is regarded as a random variable, and several situations with different types of hazard rates are examined, using the Weibull, log-normal, and Hjorth [5] distributions.

In this paper, firstly we review some previous works regarding the minimization of expected total cost per cycle: two approximate solutions for the inspection period are examined.

Secondly, the main point of this paper is presented; by minimizing expected total cost per unit of time, we obtain a new approximate solution for the inspection period. Our results are carefully analyzed for a range of different situations. System lifetime distribution is significant, as it can be seen by means of examples. As our main conclusion, the solution proposed is nearly optimal, for an extremely wide range of contexts, especially if it is considered in terms of costs. At the same time, it is easy to calculate, which is an important point when considering practical applications. In addition, this approximate solution can be regarded as a generalization of a previous solution. These two approaches approximate to each other when the relationship between costs tends towards zero.

Thirdly, when system hazard rate is constant, we propose a second solution for the inspection period, which is also extremely satisfactory. Using an exponential distribution, three approximate solutions for the expected duration of hidden faults are presented and analyzed using numerical results.

The optimum value of the inspection period may be very different depending on whether the minimization of total expected costs is carried out with respect to a cycle or to the unit of time, especially where the relative significance of inspection costs increases. However, in the opposite situation, the corresponding results are increasingly similar where the expected number of system inspections increases.

Generally, for practical purposes, it is our opinion that the new solutions put forward in this paper can be regarded as extremely satisfactory, and they are also very simple and easy to obtain. In particular, the relative errors of associated total expected costs are almost zero.

Acknowledgments

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