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Research Article

Multiple Positive Solutions for Nonlinear First-Order Impulsive Dynamic Equations on Time Scales with Parameter

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By using the Leggett-Williams fixed point theorem, the existence of three positive solutions to a class of nonlinear first-order periodic boundary value problems of impulsive dynamic equations on time scales with parameter are obtained. An example is given to illustrate the main results in this paper.

1. Introduction

Let $\mathbf{T}$ be a time scale, that is, $\mathbf{T}$ is a nonempty closed subset of R. Let $T > 0$ be fixed and $0, T$ be points in $\mathbf{T}$, an interval $(0,T)_{\mathbf{T}}$ denoting time scales interval, that is, $(0,T)_{\mathbf{T}} := (0,T) \cap \mathbf{T}$. Other types of intervals are defined similarly. Some definitions concerning time scales can be found in [1–5].

In this paper, we are concerned with the existence of positive solutions for the following nonlinear first-order periodic boundary value problem on time scales:

$$\begin{gathered} x^{\Delta}(t) + p(t)\,x(\sigma(t)) = \lambda f(t, x(\sigma(t))), \quad t \in J := [0,T]_{\mathbf{T}},\ t \neq t_k,\ k = 1,2,\ldots,m, \\ x\left(t_k^+\right) - x\left(t_k^-\right) = I_k\left(x\left(t_k^-\right)\right), \quad k = 1,2,\ldots,m, \\ x(0) = x(\sigma(T)), \end{gathered} \tag{1.1}$$

where $\lambda > 0$ is a positive parameter, $f \in C(J \times [0,\infty), [0,\infty))$, $I_k \in C([0,\infty), [0,\infty))$, $p : [0,T]_{\mathbf{T}} \to (0,\infty)$ is right-dense continuous, $t_k \in (0,T)_{\mathbf{T}}$, $0 < t_1 < \cdots < t_m < T$, and for each

$k = 1, 2, \ldots, m$, $x(t_k^+) = \lim_{h \to 0^+} x(t_k + h)$ and $x(t_k^-) = \lim_{h \to 0^-} x(t_k + h)$ represent the right and left limits of $x(t)$ at $t = t_k$.

The theory of impulsive differential equations is emerging as an important area of investigation, since it is a lot richer than the corresponding theory of differential equations without impulse effects. Moreover, such equations may exhibit several real world phenomena in physics, biology, engineering, and so forth, (see [6–8]). At the same time, the boundary value problems for impulsive differential equations and impulsive difference equations have received much attention [9–19]. On the other hand, recently, the theory of dynamic equations on time scales has become a new important branch (see, e.g., [1–5]). Naturally, some authors have focused their attention on the boundary value problems of impulsive dynamic equations on time scales [20–27]. In particular, for the first-order impulsive dynamic equations on time scales

$$\begin{gathered} y^{\Delta}(t) + p(t)\, y(\sigma(t)) = f(t, y(t)), \quad t \in J := [a, b],\ t \neq t_k,\ k = 1, 2, \ldots, m, \\ y(t_k^+) = I_k(y(t_k^-)), \quad k = 1, 2, \ldots, m, \\ y(a) = \eta, \end{gathered} \tag{1.2}$$

where $\mathbf{T}$ is a time scale which has at least finitely-many right-dense points, $[a, b] \subset \mathbf{T}$, p is regressive and right-dense continuous, $f : \mathbf{T} \times R \to R$ is given function, $I_k \in C(R, R)$. The paper [21] obtained the existence of one solution to problem (1.2) by using the nonlinear alternative of Leray-Schauder type.

In [22], Benchohra et al. considered the following impulsive boundary value problem on time scales

$$\begin{gathered} -y^{\Delta\Delta}(t) = f(t, y(t)), \quad t \in J := [0, 1]_{\mathbf{T}}, t \neq t_k, \\ y(t_k^+) - y(t_k^-) = I_k(y(t_k^-)), \\ y^{\Delta}(t_k^+) - y^{\Delta}(t_k^-) = \overline{I}_k(y(t_k^-)), \\ y(0) = y(1) = 0. \end{gathered} \tag{1.3}$$

They proved the existence of one solution to the problem (1.3) by applying Schaefer's fixed point theorem and the nonlinear alternative of Leray-Schauder type.

In [26], Li and Shen studied the problem (1.3). Some existence results to problem (1.3) are established by using a fixed point theorem, which is due to Krasnoselskii and Zabreiko, and the Leggett-Williams fixed point theorem.

In [27], the first author studied the problem (1.1) when $\lambda = 1$. The existence of positive solutions to the problem (1.1) was obtained by means of the well-known Guo-Krasnoselskii fixed point theorem.

Recently, Sun and Li [28] considered the following periodic boundary value problem:

$$\begin{gathered} x^{\Delta}(t) + p(t)\, x(\sigma(t)) = \lambda f(x(t)), \quad t \in [0, T]_{\mathbf{T}}, \\ x(0) = x(\sigma(T)). \end{gathered} \tag{1.4}$$

By using the fixed point index, some existence, multiplicity and nonexistence criteria of positive solutions to the problem (1.4) were obtained for suitable $\lambda > 0$.

Motivated by the results mentioned above, in this paper, we shall show that the problem (1.1) has at least three positive solutions for suitable $\lambda > 0$ by using the Leggett-Williams fixed point theorem [29]. We note that for the case $\lambda = 1$ and $I_k(x) \equiv 0, k = 1, 2, \ldots, m$, problem (1.1) reduces to the problem studied by [30].

In the remainder of this section, we state the following theorem, which are crucial to our proof.

Let E be a real Banach space and $K \subset E$ be a cone. A function $\alpha : K \to [0, \infty)$ is called a nonnegative continuous concave functional if α is continuous and

$$\alpha\left(tx + (1-t)y\right) \geq t\alpha(x) + (1-t)\alpha(y) \tag{1.5}$$

for all $x, y \in K$ and $t \in [0, 1]$.

Let $a, b > 0$ be constants, $K_a = \{x \in K : \|x\| < a\}, K(\alpha, a, b) = \{x \in K : a \leq \alpha(x), \|x\| \leq b\}$.

Theorem 1.1 (see [29]). *Let $A : \overline{K}_c \to \overline{K}_c$ be a completely continuous map and α be a nonnegative continuous concave functional on K such that $\alpha(x) \leq \|x\|$, $\forall x \in \overline{K}_c$. Suppose there exist a, b, d with $0 < d < a < b \leq c$ such that*

(i) $\{x \in K(\alpha, a, b) : \alpha(x) > a\} \neq \phi$ *and* $\alpha(Ax) > a \forall x \in K(\alpha, a, b)$*;*

(ii) $\|Ax\| < d \forall x \in K_d$*;*

(iii) $\alpha(Ax) > a, \forall x \in K(\alpha, a, c)$ *with* $\|Ax\| > b$*.*

Then A has at least three fixed points x_1, x_2, x_3 in $\overline{K}_c$ satisfying

$$\|x_1\| < d, \quad a < \alpha(x_2), \qquad \|x_3\| > d \quad \text{with } \alpha(x_3) < a. \tag{1.6}$$

2. Preliminaries

Throughout the rest of this paper, we always assume that the points of impulse t_k are right-dense for each $k = 1, 2, \ldots, m$.

We define

$$\begin{aligned} PC = \{x \in [0, \sigma(T)]_{\mathbf{T}} \longrightarrow R : x_k \in C(J_k, R), k = 1, 2, \ldots, m \text{ and there exist} \\ x\left(t_k^+\right) \text{ and } x\left(t_k^-\right) \text{ with } x\left(t_k^-\right) = x(t_k), k = 1, 2, \ldots, m\}, \end{aligned} \tag{2.1}$$

where x_k is the restriction of x to $J_k = (t_k, t_{k+1}]_{\mathbf{T}} \subset (0, \sigma(T)]_{\mathbf{T}}, k = 1, 2, \ldots, m$ and $J_0 = [0, t_1]_{\mathbf{T}}, J_{m+1} = \sigma(T)$.

Let

$$X = \{x(t) : x(t) \in PC, x(0) = x(\sigma(T))\} \tag{2.2}$$

with the norm $\|x\| = \sup_{t \in [0, \sigma(T)]_{\mathbf{T}}} |x(t)|$. Then X is a Banach space.

Definition 2.1. A function $x \in PC \cap C^1(J \setminus \{t_1, t_2, \dots, t_m\}, R)$ is said to be a solution of the problem (1.1) if and only if x satisfies the dynamic equation

$$x^{\Delta}(t) + p(t)x(\sigma(t)) = \lambda f(t, x(\sigma(t))) \text{ every where on } J \setminus \{t_1, t_2, \dots, t_m\}, \tag{2.3}$$

the impulsive conditions

$$x(t_k^+) - x(t_k^-) = I_k(x(t_k^-)), \quad k = 1, 2, \dots, m, \tag{2.4}$$

and the periodic boundary condition $x(0) = x(\sigma(T))$.

Lemma 2.2. *Suppose $h : [0, T]_{\mathbf{T}} \to R$ is rd-continuous, then x is a solution of*

$$x(t) = \lambda \int_0^{\sigma(T)} G(t, s) h(s) \Delta s + \sum_{k=1}^{m} G(t, t_k) I_k(x(t_k)), \quad t \in [0, \sigma(T)]_{\mathbf{T}}, \tag{2.5}$$

where

$$G(t, s) = \begin{cases} \dfrac{e_p(s, t) e_p(\sigma(T), 0)}{e_p(\sigma(T), 0) - 1}, & 0 \le s \le t \le \sigma(T), \\ \dfrac{e_p(s, t)}{e_p(\sigma(T), 0) - 1}, & 0 \le t < s \le \sigma(T), \end{cases} \tag{2.6}$$

if and only if x is a solution of the boundary value problem

$$\begin{gathered} x^{\Delta}(t) + p(t)x(\sigma(t)) = \lambda h(t), \quad t \in J := [0, T]_{\mathbf{T}},\ t \ne t_k,\ k = 1, 2, \dots, m, \\ x(t_k^+) - x(t_k^-) = I_k(x(t_k^-)), \quad k = 1, 2, \dots, m, \\ x(0) = x(\sigma(T)). \end{gathered} \tag{2.7}$$

Proof. Since the method is similar to that of in [27, Lemma 3.1], we omit it here. □

Lemma 2.3. *Let $G(t, s)$ be defined as Lemma 2.2, then*

$$\frac{1}{e_p(\sigma(T), 0) - 1} \le G(t, s) \le \frac{e_p(\sigma(T), 0)}{e_p(\sigma(T), 0) - 1} \quad \forall t,\ s \in [0, \sigma(T)]_{\mathbf{T}}. \tag{2.8}$$

Proof. It is obvious, so we omit it here. □

Let

$$K = \{x(t) \in X : x(t) \ge \delta \|x\|\}, \tag{2.9}$$

where $\delta = 1/e_p(\sigma(T), 0) \in (0, 1)$. It is not difficult to verify that K is a cone in X.

We define an operator $\Phi : K \to X$ by

$$(\Phi x)(t) = \lambda \int_0^{\sigma(T)} G(t,s) f(s, x(\sigma(s)))\Delta s + \sum_{k=1}^{m} G(t,t_k) I_k(x(t_k)), \quad t \in [0,\sigma(T)]_{\mathbf{T}}. \tag{2.10}$$

By [27, Lemmas 3.3 and 3.4], it is easy to see that $\Phi : K \to K$ is completely continuous.

3. Main Result

Notation 1. Let

$$\begin{aligned} f^0 &= \lim_{x\to 0} \sup \max_{t\in[0,T]_{\mathbf{T}}} \frac{f(t,x)}{x}, \qquad I^0 = \lim_{x\to 0} \sup \sum_{k=1}^{m} \frac{I_k(x)}{x}, \\ f^\infty &= \lim_{x\to \infty} \sup \max_{t\in[0,T]_{\mathbf{T}}} \frac{f(t,x)}{x}, \qquad I^\infty = \lim_{x\to \infty} \sup \sum_{k=1}^{m} \frac{I_k(x)}{x}, \end{aligned} \tag{3.1}$$

and for $\mu > 0$, we define $I_{(\mu)} = \min_{\delta\mu \le x \le \mu} \sum_{k=1}^{m} I_k(x)$.

Theorem 3.1. *Assume that there exists a number $b > 0$ such that the following conditions:*

(H_1) $f(t,x) > e_p(\sigma(T),0)x - e_p(\sigma(T),0)/(e_p(\sigma(T),0)-1)I_{(b)} \ge 0$ *for* $\delta b \le x \le b, t \in [0,T]_{\mathbf{T}}$;

(H_2) $f^0 + I^0 < (e_p(\sigma(T),0)-1)/e_p(\sigma(T),0)$, $f^\infty + I^\infty < (e_p(\sigma(T),0)-1)/e_p(\sigma(T),0)$ *hold. Then the problem* (1.1) *has at least three positive solutions for*

$$\frac{e_p(\sigma(T),0) - 1}{\sigma(T) e_p(\sigma(T),0)} < \lambda < \frac{1}{\sigma(T)}. \tag{3.2}$$

Proof. Let $\alpha(x) = \min_{t\in[0,\sigma(T)]_{\mathbf{T}}} x(t)$, it is easy to see that $\alpha(x)$ is a nonnegative continuous concave functional on K such that $\alpha(x) \le \|x\|, \forall x \in \overline{K}_c$.

First, we assert that there exists $c > b$ such that $\Phi : \overline{K}_c \to \overline{K}_c$ is completely continuous.

In fact, by the condition $f^\infty + I^\infty < (e_p(\sigma(T),0)-1)/e_p(\sigma(T),0)$ of (H_2), there exist $C_0 > b$, and $0 < \varepsilon < ((e_p(\sigma(T),0)-1)/e_p(\sigma(T),0) - (f^\infty + I^\infty))/2$ such that

$$f(t,x) \le (\varepsilon + f^\infty)x, \sum_{k=1}^{m} I_k(x) \le (\varepsilon + I^\infty)x, \quad \text{for } x > C_0. \tag{3.3}$$

Let $C_1 = C_0/\delta$, if $x \in K, \|x\| > C_1$, then $x > C_0$ and we have

$$\begin{aligned}(\Phi x)(t) &= \lambda \int_0^{\sigma(T)} G(t,s) f(s, x(\sigma(s)))\Delta s + \sum_{k=1}^{m} G(t,t_k) I_k(x(t_k)) \\ &\le \lambda \frac{e_p(\sigma(T),0)}{e_p(\sigma(T),0)-1}\int_0^{\sigma(T)} \left(\varepsilon + f^{\infty}\right)\|x\|\Delta s + \frac{e_p(\sigma(T),0)}{e_p(\sigma(T),0)-1}(\varepsilon + I^{\infty})\|x\| \\ &= \left[\lambda \frac{e_p(\sigma(T),0)}{e_p(\sigma(T),0)-1}\sigma(T)\left(\varepsilon + f^{\infty}\right) + \frac{e_p(\sigma(T),0)}{e_p(\sigma(T),0)-1}(\varepsilon + I^{\infty})\right]\|x\| \\ &< \|x\|.\end{aligned} \tag{3.4}$$

Take $\overline{K}_{C_1} = \{x \mid x \in K, \|x\| \le C_1\}$, then the set $\overline{K}_{C_1}$ is a bounded set. According to that Φ is completely continuous, then Φ maps bounded sets into bounded sets and there exists a number C_2 such that

$$\|\Phi x\| \le C_2 \quad \text{for any } x \in \overline{K}_{C_1}. \tag{3.5}$$

If $C_2 \le C_1$, we deduce that $\Phi : \overline{K}_{C_1} \to \overline{K}_{C_1}$ is completely continuous. If $C_1 < C_2$, then from (3.4), we know that for any $x \in \overline{K}_{C_2} \setminus \overline{K}_{C_1}, \|x\| > C_1$ and $\|\Phi x\| < \|x\| \le C_2$ hold. Then we have $\Phi : \overline{K}_{C_2} \to \overline{K}_{C_2}$ is completely continuous. Take $c = \max\{C_1, C_2\}$, then $c > b$ and $\Phi : \overline{K}_c \to \overline{K}_c$ are completely continuous.

Second, we assert that $\{x \in K(\alpha, \delta b, b) : \alpha(x) > \delta b\} \ne \phi$ and $\alpha(Ax) > \delta b$ for all $x \in K(\alpha, \delta b, b)$.

In fact, take $x \equiv (b + \delta b)/2$, so $x \in \{x \in K(\alpha, \delta b, b) : \alpha(x) > \delta b\}$. Moreover, for $x \in K(\alpha, \delta b, b)$, then $\alpha(x) \ge \delta b$ and we have

$$\begin{aligned}\alpha(\Phi x) &= \min_{t\in[0,\sigma(T)]_{\mathbb{T}}} \left[\lambda \int_0^{\sigma(T)} G(t,s) f(s, x(\sigma(s)))\Delta s + \sum_{k=1}^{m} G(t,t_k) I_k(x(t_k))\right] \\ &\ge \frac{\lambda}{e_p(\sigma(T),0)-1}\cdot \sigma(T)\left(e_p(\sigma(T),0)\,\alpha(x) - \frac{e_p(\sigma(T),0)}{e_p(\sigma(T),0)-1} I_{(b)}\right) \\ &\quad + \frac{1}{e_p(\sigma(T),0)-1} I_{(b)} \\ &> \alpha(x) \ge \delta b.\end{aligned} \tag{3.6}$$

Third, we assert that there exist $0 < d < \delta b$ such that $\|\Phi x\| < d$ if $x \in K_d$.

Indeed, by the condition $f^0 + I^0 < (e_p(\sigma(T),0) - 1)/e_p(\sigma(T),0)$ of (H_2), there exist $0 < d < \delta b$, and $0 < \varepsilon < ((e_p(\sigma(T),0) - 1)/e_p(\sigma(T),0) - (f^0 + I^0))/2$ such that

$$f(t,x) \le \left(\varepsilon + f^0\right)x, \sum_{k=1}^{m} I_k(x) \le \left(\varepsilon + I^0\right)x, \quad \text{for } 0 \le x \le d. \tag{3.7}$$

Then $x \in K_d$, we get

$$
\begin{aligned}
(\Phi x)(t) &= \lambda \int_0^{\sigma(T)} G(t,s) f(s, x(\sigma(s))) \Delta s + \sum_{k=1}^{m} G(t,t_k) I_k(x(t_k)) \\
&\leq \lambda \frac{e_p(\sigma(T),0)}{e_p(\sigma(T),0)-1} \int_0^{\sigma(T)} \left(\varepsilon + f^0\right) x(s) \Delta s + \frac{e_p(\sigma(T),0)}{e_p(\sigma(T),0)-1} \left(\varepsilon + I^0\right) \|x\| \\
&\leq \left[\lambda \frac{e_p(\sigma(T),0)}{e_p(\sigma(T),0)-1} \left(\varepsilon + f^0\right) \sigma(T) + \frac{e_p(\sigma(T),0)}{e_p(\sigma(T),0)-1} \left(\varepsilon + I^0\right)\right] \|x\| \\
&< \frac{e_p(\sigma(T),0)}{e_p(\sigma(T),0)-1} \left(f^0 + I^0 + 2\varepsilon\right) \|x\| \\
&< \|x\| < d.
\end{aligned}
\tag{3.8}
$$

Finally, we assert that $\alpha(\Phi x) > \delta b$ if $x \in K(\alpha, \delta b, c)$ and $\|\Phi x\| > b$.
To do this, if $x \in K(\alpha, \delta b, c)$ and $\|\Phi x\| > b$, then

$$
\alpha(\Phi x) \geq (\Phi x)(t) \geq \delta \|\Phi x\| > \delta b. \tag{3.9}
$$

To sum up, all the hypotheses of Theorem 1.1 are satisfied by taking $a = \delta b$. Hence Φ has at least three fixed points, that is, the problem (1.1) has at least three positive solutions x_1, x_2 and x_3 such that

$$
\|x_1\| < d, a < \alpha(x_2), \quad \|x_3\| > d \text{ with } \alpha(x_3) < a. \tag{3.10}
$$

□

Corollary 3.2. *Using (H_3) $f^0 = I^0 = f^\infty = I^\infty = 0$, instead of ($H_2$) in Theorem 3.1, the conclusion of Theorem 3.1 remains true.*

4. Example

Example 4.1. Let $\mathbf{T} = [0,1] \cup [2,3]$. We consider the following problem on $\mathbf{T}$:

$$
\begin{gathered}
x^{\Delta}(t) + x(\sigma(t)) = \lambda f(t, x(\sigma(t))), \quad t \in [0,3]_{\mathbf{T}},\ t \neq \frac{1}{2}, \\
x\left(\frac{1}{2}^{+}\right) - x\left(\frac{1}{2}^{-}\right) = I\left(x\left(\frac{1}{2}\right)\right), \\
x(0) = x(3),
\end{gathered}
\tag{4.1}
$$

where $\lambda > 0$ is a positive parameter, $p(t) \equiv 1, T = 3, m = 1$, and

$$f(t,x) = \begin{cases} 9e^6(t+1)x^2, & [0,1], \\ 9e^6(t+1)x^{1/2}, & [1,\infty), \end{cases}$$
$$I(x) = \begin{cases} x^2, & [0,1], \\ x^{1/2}, & [1,\infty). \end{cases} \tag{4.2}$$

Taking $b = 1$, then by $\delta = 1/(2e^2)$ it is easy to see that $I_{(b)} = \min_{\delta b \le x \le b} I(x) = 1/(4e^4)$. So, $\forall x \in [\delta b, b] = [1/(2e^2), 1]$, we have $f(t,x) \ge (9/4e^2) > 2e^2 - 1/[(2e^2-1)2e^2] \ge 2e^2 x - (2e^2)/(2e^2-1)1/(4e^4) = e_p(\sigma(T),0)x - e_p(\sigma(T),0)/(e_p(\sigma(T),0)-1)I_{(b)}$. Obviously, we have $f^0 = I^0 = f^\infty = I^\infty = 0$.

Therefore, together with Corollary 3.2, it follows that the problem (4.1) has at least three positive solutions for $(2e^2-1)/(6e^2) < \lambda < 1/3$.

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