Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2008, Article ID 296159, 10 pages
doi:10.1155/2008/296159

Research Article

On the q-Extension of Apostol-Euler Numbers and Polynomials

Young-Hee Kim,[1] Wonjoo Kim,[2] and Lee-Chae Jang[3]

[1] *Division of General Education-Mathematics, Kwangwoon University, Seoul 139-701, South Korea*
[2] *Natural Science Institute, KonKuk University, Chungju 380-701, South Korea*
[3] *Department of Mathematics and Computer Science, KonKuk University, Chungju 380-701, South Korea*

Correspondence should be addressed to Lee-Chae Jang, leechae.jang@kku.ac.kr

Received 4 October 2008; Accepted 21 November 2008

Recommended by Lance Littlejohn

Recently, Choi et al. (2008) have studied the q-extensions of the Apostol-Bernoulli and the Apostol-Euler polynomials of order n and multiple Hurwitz zeta function. In this paper, we define Apostol's type q-Euler numbers $E_{n,q,\xi}$ and q-Euler polynomials $E_{n,q,\xi}(x)$. We obtain the generating functions of $E_{n,q,\xi}$ and $E_{n,q,\xi}(x)$, respectively. We also have the distribution relation for Apostol's type q-Euler polynomials. Finally, we obtain q-zeta function associated with Apostol's type q-Euler numbers and Hurwitz's type q-zeta function associated with Apostol's type q-Euler polynomials for negative integers.

1. Introduction

Let p be a fixed odd prime. Throughout this paper, $\mathbb{Z}_p$, $\mathbb{Q}_p$ and $\mathbb{C}$ and $\mathbb{C}_p$ will, respectively, denote the ring of p-adic rational integers, the field of p-adic rational numbers, the complex number field, and the completion of algebraic closure of $\mathbb{Q}_p$. Let $\mathbb{N}$ be the set of natural numbers and $\mathbb{Z}_+ = \mathbb{N} \cup \{0\}$. Let v_p be the normalized exponential valuation of $\mathbb{C}_p$ with $|p|_p = p^{-v_p(p)} = p^{-1}$. When one talks of q-extension, q is variously considered as an indeterminate, a complex number $q \in \mathbb{C}$, or a p-adic number $q \in \mathbb{C}_p$. If $q \in \mathbb{C}$, one normally assumes $|q| < 1$. If $q \in \mathbb{C}_p$, then one assumes $|q-1|_p < 1$. We also use the notations

$$[x]_q = \frac{1-q^x}{1-q}, \qquad [x]_{-q} = \frac{1-(-q)^x}{1+q} \quad \forall x \in \mathbb{Z}_p \tag{1.1}$$

For a fixed odd positive integer d with $(p,d)=1$, let

$$X = X_d = \lim_{\overleftarrow{N}} \frac{\mathbb{Z}}{dp^N\mathbb{Z}}, \qquad X_1 = \mathbb{Z}_p,$$

$$X^* = \bigcup_{\substack{0<a<dp \\ (a,p)=1}} (a + dp\mathbb{Z}_p),$$

$$a + dp^N\mathbb{Z}_p = \{x \in X \mid x \equiv a (\bmod\ dp^N)\}, \tag{1.2}$$

where $a \in \mathbb{Z}$ lies in $0 \leq a < dp^N$. The distribution is defined by

$$\mu_q(a + dp^N\mathbb{Z}_p) = \frac{q^a}{[dp^N]_q}. \tag{1.3}$$

We say that f is a uniformly differentiable function at a point $a \in \mathbb{Z}_p$ and denote this property by $f \in \mathrm{UD}(\mathbb{Z}_p)$, if the difference quotients $F_f(x, y) = (f(x) - f(y))/(x - y)$ have a limit $l = f'(a)$ as $(x, y) \to (a, a)$. For $f \in \mathrm{UD}(\mathbb{Z}_p)$, the p-adic invariant q-integral is defined as

$$I_q(f) = \int_{\mathbb{Z}_p} f(x)d\mu_q(x) = \lim_{N\to\infty} \frac{1}{[p^N]_q} \sum_{x=0}^{p^N-1} f(x)q^x. \tag{1.4}$$

The fermionic p-adic q-measures on $\mathbb{Z}_p$ are defined as

$$\mu_{-q}(a + dp^N\mathbb{Z}_p) = \frac{(-q)^a}{[dp^N]_{-q}}, \tag{1.5}$$

and the fermionic p-adic invariant q-integral on $\mathbb{Z}_p$ is defined as

$$I_{-q}(f) = \int_{\mathbb{Z}_p} f(x)d\mu_{-q}(x) = \lim_{N\to\infty} \frac{1}{[p^N]_{-q}} \sum_{x=0}^{p^N-1} f(x)(-q)^x \tag{1.6}$$

for $f \in \mathrm{UD}(\mathbb{Z}_p)$. For details see [1–10].

Classical Euler numbers are defined by the generating function

$$\frac{2}{e^t + 1} = \sum_{n=0}^{\infty} E_n \frac{t^n}{n!}, \tag{1.7}$$

and these numbers are interpolated by the Euler zeta function which is defined as

$$\zeta_E(s) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n^s}, \quad s \in \mathbb{C}. \tag{1.8}$$

After Carlitz [11] gave q-extensions of the classical Bernoulli numbers and polynomials, the q-extensions of Bernoulli and Euler numbers and polynomials have been studied by several authors (cf. [1–16, 18–26, 34–39]).

By using p-adic q-integral, the q-Euler numbers $E_{n,q}$ are defined as

$$E_{n,q} = \int_{\mathbb{Z}_p} [t]_q^n d\mu_{-q}(t), \quad \text{for } n \in \mathbb{N}. \tag{1.9}$$

The q-Euler numbers $E_{n,q}$ are defined by means of the generating function

$$F_q(t) = [2]_q \sum_{n=0}^{\infty} (-1)^n q^n e^{[n]_q t} \tag{1.10}$$

(cf. [8, 26]). Kim [22] gave a new construction of the q-Euler numbers $E_{n,q}$ which can be uniquely determined by

$$E_{0,q} = \frac{[2]_q}{2},$$
$$(qE+1)^n + E_{n,q} = \begin{cases} [2]_q, & \text{if } n = 0, \\ 0, & \text{if } n \neq 0, \end{cases} \tag{1.11}$$

with the usual convention of replacing E^n by $E_{n,q}$.

The twisted q-Euler numbers and q-Euler polynomials are very important in several fields of mathematics and physics, and so they have been studied by many authors. Simsek [37, 38] constructed generating functions of q-generalized Euler numbers and polynomials and twisted q-generalized Euler numbers and polynomials. Recently, Y. H. Kim et al. [27] gave the twisted q-Euler zeta function associated with twisted q-Euler numbers and obtained q-Euler's identity. They also have a q-extension of the Euler zeta function for negative integers and the q-analog of twisted Euler zeta function. Kim [24] defined twisted q-Euler numbers and polynomials of higher order and studied multiple twisted q-Euler zeta functions.

The Apostol-Bernoulli and the Apostol-Euler polynomials and numbers have been studied by several authors (cf. [15, 17, 32, 33, 40, 41]). Recently, q-extensions of the Apostol-Bernoulli and the Apostol-Euler polynomials and numbers have been studied by many authors with great interest. In [15], Cenkci and Can introduced and investigated q-extensions of the Bernoulli polynomials. Choi et al. [16] have studied some q-extensions of the Apostol-Bernoulli and the Apostol-Euler polynomials of order n and multiple Hurwitz zeta function.

In this paper, we define Apostol's type q-Euler numbers and q-Euler polynomials. Then, we have the generating functions of Apostol's type q-Euler numbers and q-Euler polynomials and the distribution relation for Apostol's type q-Euler polynomials. In Section 2, we define Apostol's type q-Euler numbers $E_{n,q,\xi}$ and q-Euler polynomials $E_{n,q,\xi}(x)$. Then, we obtain the generating functions of $E_{n,q,\xi}$ and $E_{n,q,\xi}(x)$, respectively. We also have the distribution relation for Apostol's type q-Euler polynomials. In Section 3, we obtain q-zeta function associated with Apostol's type q-Euler numbers and Hurwitz's type q-zeta function associated with Apostol's type q-Euler polynomials for negative integers.

2. On the q-extensions of the Apostol-Euler numbers and polynomials

In this section, we will assume $q \in \mathbb{C}_p$ with $|q-1|_p < 1$. For $n \in \mathbb{Z}_+$, let $C_{p^n} = \{\xi \mid \xi^{p^n} = 1\}$ be the cyclic group of order p^n, and let T_p be the space of locally constant space, that is,

$$T_p = \lim_{n\to\infty} C_{p^n} = \bigcup_{n\geq 0} C_{p^n}. \tag{2.1}$$

Let $\xi \in T_p$. We define Apostol's type q-Euler numbers by

$$E_{n,q,\xi} = \int_{\mathbb{Z}_p} q^{-x}\xi^x [x]_q^n d\mu_{-q}(x). \tag{2.2}$$

Then, we have

$$E_{n,q,\xi} = \frac{[2]_q}{(1-q)^n} \sum_{l=0}^{n} \binom{n}{l} (-1)^l \frac{1}{1+q^l\xi}, \tag{2.3}$$

where $\binom{n}{l}$ are the binomial coefficients.

Apostol's type q-Euler polynomials are defined as

$$E_{n,q,\xi}(x) = \int_{\mathbb{Z}_p} q^{-y}\xi^y [x+y]_q^n d\mu_{-q}(y). \tag{2.4}$$

Since

$$[x+y]_q^n = \left([x]_q + q^x[y]_q\right)^n = \sum_{l=0}^{n} \binom{n}{l} [x]_q^{n-l} q^{lx} [y]_q^l, \tag{2.5}$$

we have from (2.4) that

$$E_{n,q,\xi}(x) = \sum_{l=0}^{n} \binom{n}{l} [x]_q^{n-l} q^{lx} \int_{\mathbb{Z}_p} q^{-y}\xi^y [y]_q^l d\mu_{-q}(y). \tag{2.6}$$

By (2.2) and (2.6), we have

$$E_{n,q,\xi}(x) = \sum_{l=0}^{n} \binom{n}{l} [x]_q^{n-l} q^{lx} E_{l,q,\xi}. \tag{2.7}$$

Since

$$[x+y]_q^n = \frac{1}{(1-q)^n} \sum_{l=0}^{n} \binom{n}{l} (-1)^l q^{(x+y)l} = \frac{1}{(1-q)^n} \sum_{l=0}^{n} \binom{n}{l} (-1)^l q^{lx} q^{ly}, \tag{2.8}$$

we have

$$\int_{\mathbb{Z}_p} q^{-y}\xi^y [x+y]_q^n d\mu_{-q}(y) = \frac{1}{(1-q)^n}\sum_{l=0}^{n}\binom{n}{l}(-1)^l q^{lx}\int_{\mathbb{Z}_p} q^{(l-1)y}\xi^y d\mu_{-q}(y). \tag{2.9}$$

Therefore, we also have

$$E_{n,q,\xi}(x) = [2]_q \frac{1}{(1-q)^n}\sum_{l=0}^{n}\binom{n}{l} q^{lx}(-1)^l \frac{1}{1+q^l\xi}. \tag{2.10}$$

Note that (2.7) and (2.10) are two representations for $E_{n,q,\xi}(x)$. Hence, we have the following result.

Theorem 2.1. *For $n \in \mathbb{Z}_+$ and $\xi \in T_p$, one has*

$$\begin{aligned} E_{n,q,\xi} &= \frac{[2]_q}{(1-q)^n}\sum_{l=0}^{n}\binom{n}{l}(-1)^l \frac{1}{1+q^l\xi}, \\ E_{n,q,\xi}(x) &= \frac{[2]_q}{(1-q)^n}\sum_{l=0}^{n}\binom{n}{l}(-1)^l \frac{q^{lx}}{1+q^l\xi} \\ &= \sum_{l=0}^{n}\binom{n}{l}[x]_q^{n-l} q^{lx} E_{l,q,\xi}. \end{aligned} \tag{2.11}$$

Now, we will find the generating function of $E_{n,q,\xi}$ and $E_{n,q,\xi}(x)$, respectively. Let $F(t)$ be the generating function of $E_{n,q,\xi}$. Then, we have

$$\begin{aligned} F(t) &= \sum_{n=0}^{\infty} E_{n,q,\xi}\frac{t^n}{n!} \\ &= \sum_{n=0}^{\infty}\left(\frac{[2]_q}{(1-q)^n}\sum_{l=0}^{n}\binom{n}{l}(-1)^l\frac{1}{1+q^l\xi}\right)\frac{t^n}{n!} \\ &= [2]_q\sum_{n=0}^{\infty}\frac{1}{(1-q)^n}\sum_{l=0}^{n}\binom{n}{l}(-1)^l\left(\sum_{m=0}^{\infty} q^{lm}\xi^m(-1)^m\right)\frac{t^n}{n!} \\ &= [2]_q\sum_{m=0}^{\infty}(-1)^m\xi^m\sum_{n=0}^{\infty}\frac{1}{(1-q)^n}\left(\sum_{l=0}^{n}\binom{n}{l}(-1)^l q^{lm}\right)\frac{t^n}{n!} \\ &= [2]_q\sum_{m=0}^{\infty}(-1)^m\xi^m\sum_{n=0}^{\infty}\frac{1}{(1-q)^n}(1-q^m)^n\frac{t^n}{n!} \end{aligned}$$

$$= [2]_q \sum_{m=0}^{\infty} (-1)^m \xi^m \sum_{n=0}^{\infty} [m]_q^n \frac{t^n}{n!}$$
$$= [2]_q \sum_{m=0}^{\infty} (-1)^m \xi^m e^{[m]_q t}. \tag{2.12}$$

Therefore, the generating function $F(t)$ of $E_{n,q,\xi}$ equals

$$F(t) = \sum_{n=0}^{\infty} E_{n,q,\xi} \frac{t^n}{n!} = [2]_q \sum_{m=0}^{\infty} (-1)^m \xi^m e^{[m]_q t}. \tag{2.13}$$

Note that

$$\int_{\mathbb{Z}_p} q^{-x} \xi^x e^{[x]_q t} \, d\mu_{-q}(x) = \sum_{n=0}^{\infty} \int_{\mathbb{Z}_p} q^{-x} \xi^x [x]_q^n \, d\mu_{-q}(x) \frac{t^n}{n!}$$
$$= \sum_{n=0}^{\infty} E_{n,q,\xi} \frac{t^n}{n!} = F(t). \tag{2.14}$$

For the generating function of $E_{n,q,\xi}(x)$, we have

$$\int_{\mathbb{Z}_p} q^{-y} \xi^y e^{[x+y]_q t} \, d\mu_{-q}(y) = [2]_q \sum_{m=0}^{\infty} (-1)^m \xi^m e^{[m+x]_q t}. \tag{2.15}$$

Hence, we obtain the following theorem.

Theorem 2.2. *For $\xi \in T_p$, one has*

$$\int_{\mathbb{Z}_p} q^{-x} \xi^x e^{[x]_q t} \, d\mu_{-q}(x) = [2]_q \sum_{m=0}^{\infty} (-1)^m \xi^m e^{[m]_q t}, \tag{2.16}$$

$$\int_{\mathbb{Z}_p} q^{-y} \xi^y e^{[x+y]_q t} \, d\mu_{-q}(y) = [2]_q \sum_{m=0}^{\infty} (-1)^m \xi^m e^{[m+x]_q t}. \tag{2.17}$$

Since (2.16) equals to the generating functions (2.17) equals to the generating functions $\sum_{n=0}^{\infty} E_{n,q,\xi}(x)(t^n/n!)$, we have the following result.

Corollary 2.3. *For $n \in \mathbb{Z}_+$ and $\xi \in T_p$, one has*

$$E_{n,q,\xi} = [2]_q \sum_{m=0}^{\infty} (-1)^m \xi^m [m]_q^n,$$
$$E_{n,q,\xi}(x) = [2]_q \sum_{m=0}^{\infty} (-1)^m \xi^m [m+x]_q^n. \tag{2.18}$$

Now, we will find the distribution relation for $E_{n,q,\xi}(x)$. By (2.4), we have

$$\begin{aligned} E_{n,q,\xi}(x) &= \int_X q^{-y}\xi^y [x+y]_q^n \, d\mu_{-q}(y) \\ &= \lim_{N\to\infty} \frac{1}{[dp^N]_{-q}} \sum_{y=0}^{dp^N-1} \xi^y (-1)^y [x+y]_q^n \\ &= \lim_{N\to\infty} \frac{1}{[dp^N]_{-q}} \sum_{a=0}^{d-1}\sum_{y=0}^{p^N-1} \xi^{a+dy} (-1)^{a+dy} [x+a+dy]_q^n. \end{aligned} \tag{2.19}$$

Note that for odd numbers d and p,

$$\begin{aligned} [dp^N]_{-q} &= [d]_{-q}[p^N]_{-q^d}, \\ [x+a+dy]_q &= [d]_q \left[\frac{x+a}{d} + y\right]_{q^d}. \end{aligned} \tag{2.20}$$

By (2.19), we have

$$\begin{aligned} E_{n,q,\xi}(x) &= \frac{1}{[d]_{-q}} \sum_{a=0}^{d-1} \xi^a (-1)^a \lim_{N\to\infty} \frac{1}{[p^N]_{-q^d}} \sum_{y=0}^{p^N-1} (\xi^d)^y (-1)^y [d]_q^n \left[\frac{x+a}{d} + y\right]_{q^d}^n \\ &= \frac{[d]_q^n}{[d]_{-q}} \sum_{a=0}^{d-1} \xi^a (-1)^a \int_{\mathbb{Z}_p} (\xi^d)^y (q^d)^{-y} \left[\frac{x+a}{d} + y\right]_{q^d}^n d\mu_{-q^d}(y). \end{aligned} \tag{2.21}$$

Therefore, we obtain the distribution relation for $E_{n,q,\xi}(x)$ as follows.

Theorem 2.4. *For $n \in \mathbb{Z}_+$, $\xi \in T_p$, and $d \in \mathbb{Z}_+$ with $d \equiv 1 (\mathrm{mod}\ 2)$, one has*

$$E_{n,q,\xi}(x) = \frac{[d]_q^n}{[d]_{-q}} \sum_{a=0}^{d-1} \xi^a (-1)^a E_{n,q^d,\xi^d}\left(\frac{x+a}{d}\right). \tag{2.22}$$

3. Further remark on the basic q-zeta functions associated with Apostol's type q-Euler numbers and polynomials

In this section, we assume that $q \in \mathbb{C}$ with $|q| < 1$. Let $\xi \in T_p$. For $s \in \mathbb{C}$, q-zeta function associated with Apostol's type q-Euler numbers is defined as

$$\zeta_{q,\xi}(s) = [2]_q \sum_{n=1}^{\infty} \frac{\xi^n (-1)^n}{[n]_q^s}, \tag{3.1}$$

which is analytic in whole complex s-plane. Substituting $s = -k$ with $k \in \mathbb{Z}_+$ into $\zeta_{q,\xi}(s)$ and using Corollary 2.3, then we arrive at

$$\zeta_{q,\xi}(-k) = [2]_q \sum_{n=1}^{\infty} \xi^n (-1)^n [n]_q^k = E_{k,q,\xi}. \tag{3.2}$$

Now, we also consider Hurwitz's type q-zeta function associated with the Apostol's type q-Euler polynomials as follows:

$$\zeta_{q,\xi}(s,x) = [2]_q \sum_{n=0}^{\infty} \frac{\xi^n (-1)^n}{[n+x]_q^s}. \tag{3.3}$$

Substituting $s = -k$ with $k \in \mathbb{Z}_+$ into $\zeta_{q,\xi}(s,x)$ and using Corollary 2.3, then we arrive at

$$\zeta_{q,\xi}(-k,x) = [2]_q \sum_{n=0}^{\infty} \xi^n (-1)^n [n+x]_q^k = E_{k,q,\xi}(x). \tag{3.4}$$

Hence, we obtain q-zeta function associated with Apostol's type q-Euler numbers and Hurwitz's type q-zeta function associated with Apostol's type q-Euler polynomials for negative integers.

References

[1] T. Kim, "q-Volkenborn integration," *Russian Journal of Mathematical Physics*, vol. 9, no. 3, pp. 288–299, 2002.

[2] T. Kim, "On p-adic q-L-functions and sums of powers," *Discrete Mathematics*, vol. 252, no. 1–3, pp. 179–187, 2002.

[3] T. Kim, "On Euler-Barnes multiple zeta functions," *Russian Journal of Mathematical Physics*, vol. 10, no. 3, pp. 261–267, 2003.

[4] T. Kim, "Sums of powers of consecutive q-integers," *Advanced Studies in Contemporary Mathematics*, vol. 9, no. 1, pp. 15–18, 2004.

[5] T. Kim, "Analytic continuation of multiple q-zeta functions and their values at negative integers," *Russian Journal of Mathematical Physics*, vol. 11, no. 1, pp. 71–76, 2004.

[6] T. Kim, "q-Riemann zeta function," *International Journal of Mathematics and Mathematical Sciences*, vol. 2004, no. 12, pp. 599–605, 2004.

[7] T. Kim, "Power series and asymptotic series associated with the q-analog of the two-variable p-adic L-function," *Russian Journal of Mathematical Physics*, vol. 12, no. 2, pp. 186–196, 2005.

[8] T. Kim, "q-generalized Euler numbers and polynomials," *Russian Journal of Mathematical Physics*, vol. 13, no. 3, pp. 293–298, 2006.

[9] T. Kim, "Multiple p-adic L-function," *Russian Journal of Mathematical Physics*, vol. 13, no. 2, pp. 151–157, 2006.

[10] T. Kim, "On the analogs of Euler numbers and polynomials associated with p-adic q-integral on $\mathbb{Z}_p$ at $q = -1$," *Journal of Mathematical Analysis and Applications*, vol. 331, no. 2, pp. 779–792, 2007.

[11] L. Carlitz, "q-Bernoulli numbers and polynomials," *Duke Mathematical Journal*, vol. 15, no. 4, pp. 987–1000, 1948.

[12] I. N. Cangul, H. Ozden, and Y. Simsek, "Generating functions of the (h,q) extension of twisted Euler polynomials and numbers," *Acta Mathematica Hungarica*, vol. 120, no. 3, pp. 281–299, 2008.

[13] L. Carlitz, "q-Bernoulli and Eulerian numbers," *Transactions of the American Mathematical Society*, vol. 76, no. 2, pp. 332–350, 1954.

[14] M. Cenkci, "The p-adic generalized twisted (h, q)-Euler-l-function and its applications," *Advanced Studies in Contemporary Mathematics*, vol. 15, no. 1, pp. 37–47, 2007.

[15] M. Cenkci and M. Can, "Some results on q-analogue of the Lerch zeta function," *Advanced Studies in Contemporary Mathematics*, vol. 12, no. 2, pp. 213–223, 2006.

[16] J. Choi, P. J. Anderson, and H. M. Srivastava, "Some q-extensions of the Apostol-Bernoulli and the Apostol-Euler polynomials of order n, and the multiple Hurwitz zeta function," *Applied Mathematics and Computation*, vol. 199, no. 2, pp. 723–737, 2008.

[17] M. Garg, K. Jain, and H. M. Srivastava, "Some relationships between the generalized Apostol-Bernoulli polynomials and Hurwitz-Lerch zeta functions," *Integral Transforms and Special Functions*, vol. 17, no. 11, pp. 803–815, 2006.

[18] L.-C. Jang, "Multiple twisted q-Euler numbers and polynomials associated with p-adic q-integrals," *Advances in Difference Equations*, vol. 2008, Article ID 738603, 11 pages, 2008.

[19] T. Kim, "On the q-extension of Euler and Genocchi numbers," *Journal of Mathematical Analysis and Applications*, vol. 326, no. 2, pp. 1458–1465, 2007.

[20] T. Kim, "q-extension of the Euler formula and trigonometric functions," *Russian Journal of Mathematical Physics*, vol. 14, no. 3, pp. 275–278, 2007.

[21] T. Kim, "On the symmetries of the q-Bernoulli polynomials," *Abstract and Applied Analysis*, vol. 2008, Article ID 914367, 7 pages, 2008.

[22] T. Kim, "The modified q-Euler numbers and polynomials," *Advanced Studies in Contemporary Mathematics*, vol. 16, no. 2, pp. 161–170, 2008.

[23] T. Kim, "q-Bernoulli numbers and polynomials associated with Gaussian binomial coefficients," *Russian Journal of Mathematical Physics*, vol. 15, no. 1, pp. 51–57, 2008.

[24] T. Kim, "On the multiple q-Genocchi and Euler numbers," *Russian Journal of Mathematical Physics*, vol. 15, no. 4, pp. 481–486, 2008.

[25] T. Kim, J. Y. Choi, and J. Y. Sug, "Extended q-Euler numbers and polynomials associated with fermionic p-adic q-integral on $\mathbb{Z}_p$," *Russian Journal of Mathematical Physics*, vol. 14, no. 2, pp. 160–163, 2007.

[26] T. Kim, M.-S. Kim, L. Jang, and S.-H. Rim, "New q-Euler numbers and polynomials associated with p-adic q-integrals," *Advanced Studies in Contemporary Mathematics*, vol. 15, no. 2, pp. 243–252, 2007.

[27] Y. H. Kim, W. J. Kim, and C. S. Ryoo, "On the twisted q-Euler zeta function associated with twisted q-Euler numbers," communicated.

[28] T. Kim, S.-H. Rim, and Y. Simsek, "A note on the alternating sums of powers of consecutive q-integers," *Advanced Studies in Contemporary Mathematics*, vol. 13, no. 2, pp. 159–164, 2006.

[29] T. Kim and Y. Simsek, "Analytic continuation of the multiple Daehee q-l-functions associated with Daehee numbers," *Russian Journal of Mathematical Physics*, vol. 15, no. 1, pp. 58–65, 2008.

[30] S.-D. Lin and H. M. Srivastava, "Some families of the Hurwitz-Lerch zeta functions and associated fractional derivative and other integral representations," *Applied Mathematics and Computation*, vol. 154, no. 3, pp. 725–733, 2004.

[31] S.-D. Lin, H. M. Srivastava, and P.-Y. Wang, "Some expansion formulas for a class of generalized Hurwitz-Lerch zeta functions," *Integral Transforms and Special Functions*, vol. 17, no. 11, pp. 817–827, 2006.

[32] Q.-M. Luo, "Apostol-Euler polynomials of higher order and Gaussian hypergeometric functions," *Taiwanese Journal of Mathematics*, vol. 10, no. 4, pp. 917–925, 2006.

[33] Q.-M. Luo and H. M. Srivastava, "Some generalizations of the Apostol-Bernoulli and Apostol-Euler polynomials," *Journal of Mathematical Analysis and Applications*, vol. 308, no. 1, pp. 290–302, 2005.

[34] H. Ozden, I. N. Cangul, and Y. Simsek, "Multivariate interpolation functions of higher-order q-Euler numbers and their applications," *Abstract and Applied Analysis*, vol. 2008, Article ID 390857, 16 pages, 2008.

[35] H. Ozden, I. N. Cangul, and Y. Simsek, "Remarks on sum of products of (h, q)-twisted Euler polynomials and numbers," *Journal of Inequalities and Applications*, vol. 2008, Article ID 816129, 8 pages, 2008.

[36] H. Ozden and Y. Simsek, "A new extension of q-Euler numbers and polynomials related to their interpolation functions," *Applied Mathematics Letters*, vol. 21, no. 9, pp. 934–939, 2008.

[37] Y. Simsek, "On p-adic twisted q-L-functions related to generalized twisted Bernoulli numbers," *Russian Journal of Mathematical Physics*, vol. 13, no. 3, pp. 340–348, 2006.

[38] Y. Simsek, "Generating functions of the twisted Bernoulli numbers and polynomials associated with their interpolation functions," *Advanced Studies in Contemporary Mathematics*, vol. 16, no. 2, pp. 251–278, 2008.

[39] H. M. Srivastava, T. Kim, and Y. Simsek, "q-Bernoulli numbers and polynomials associated with multiple q-zeta functions and basic L-series," *Russian Journal of Mathematical Physics*, vol. 12, no. 2, pp. 241–268, 2005.

[40] T. M. Apostol, "On the Lerch zeta function," *Pacific Journal of Mathematics*, vol. 1, pp. 161–167, 1951.

[41] W. Wang, C. Jia, and T. Wang, "Some results on the Apostol-Bernoulli and Apostol-Euler polynomials," *Computers & Mathematics with Applications*, vol. 55, no. 6, pp. 1322–1332, 2008.