

SOBOLEV ORTHOGONAL POLYNOMIALS: INTERPOLATION AND APPROXIMATION *

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Abstract. In this paper, we study orthogonal polynomials with respect to the bilinear form

$$(f, g)_S = (f(c_0), f(c_1), \dots, f(c_{N-1}))\mathbf{A} \begin{pmatrix} g(c_0) \\ g(c_1) \\ \vdots \\ g(c_{N-1}) \end{pmatrix} + \langle u, f^{(N)}g^{(N)} \rangle,$$

where u is a quasi-definite (or regular) linear functional on the linear space \mathbb{P} of real polynomials, c_0, c_1, \dots, c_{N-1} are distinct real numbers, N is a positive integer number, and \mathbf{A} is a real $N \times N$ matrix such that each of its principal submatrices are nonsingular. We show a connection between these non-standard orthogonal polynomials and some standard problems in the theory of interpolation and approximation.

Key words. Sobolev orthogonal polynomials, classical orthogonal polynomials, interpolation, approximation.

AMS subject classifications. 33C45, 42C05.

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