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$Q-$CLASSICAL ORTHOGONAL POLYNOMIALS: A VERY CLASSICAL APPROACH*

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Abstract. The $q-$classical orthogonal polynomials defined by Hahn satisfy a Sturm-Liouville type equation in geometric differences. Working with this, we classify the $q-$classical polynomials in twelve families according to the zeros of the polynomial coefficients of the equation and the behavior concerning to q^{-1}. We determine a $q-$analogue of the weight function for the twelve families, and we give a representation of its orthogonality relation and its $q-$integral. We describe this representation in some normal and special cases (indeterminate moment problem and finite orthogonal sequences). Finally, the Sturm-Liouville type equation allows us to establish the correspondence between this classification and the Askey Scheme.

Key words. orthogonal $q-$polynomials, classical polynomials.

AMS subject classifications. 33D25.

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