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ON THE CONVERGENCE OF MULTIGRID METHODS FOR FLOW PROBLEMS*

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Abstract. We prove two theorems on the residual damping in multigrid methods when solving convection dominated diffusion equations and shock wave problems, discretized by the streamline diffusion finite element method. The first theorem shows that a V-cycle, including sufficiently many pre and post smoothing steps, damps the residual in L_1^{loc} for a constant coefficient convection problem with small diffusion in two space dimensions, without the assumption that the coarse grid is sufficiently fine. The proof is based on discrete Green's functions for the smoothing and correction operators on a uniform unbounded mesh aligned with the characteristic. The second theorem proves a similar result for a certain continuous version of a two grid method, with isotropic artificial diffusion, applied to a two dimensional Burgers shock wave problem. We also present numerical experiments that verify the residual damping dependence on the equation, the choice of artificial diffusion and the number of smoothing steps. In particular numerical experiments show improved convergence of the multigrid method, with damped Jacobi smoothing steps, for the compressible Navier-Stokes equations in two space dimensions by using the theoretically suggested exponential increase of the number of smoothing steps on coarser meshes, as compared to the same amount of work with constant number of smoothing steps on each level.

Key words. multigrid methods, convergence, convection-diffusion, conservation laws, Green's function, shock waves.

AMS subject classifications. 65N55, 65N30, 35L65.

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