

PRECONDITIONERS FOR LEAST SQUARES PROBLEMS BY LU FACTORIZATION*

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Abstract. Iterative methods are often suitable for solving least-squares problems $\min \|Ax - b\|_2$, where $A \in \mathbf{R}^{m \times n}$ is large and sparse. The use of the conjugate gradient method with a nonsingular square submatrix $A_1 \in \mathbf{R}^{n \times n}$ of A as preconditioner was first suggested by Läuchli in 1961. This conjugate gradient method has recently been extended by Yuan to generalized least-squares problems.

In this paper we consider the problem of finding a suitable submatrix A_1 and its LU factorization for a sparse rectangular matrix A . We give three algorithms based on the sparse LU factorization algorithm by Gilbert and Peierls.

Numerical results are given, which indicate that our preconditioners can be effective.

Key words. Linear least squares, preconditioner, conjugate gradient method, LU factorization.

AMS subject classifications. 65F10, 65F20.

* Received November 30, 1997. Accepted for publication December 11, 1998. Recommended by D. Calvetti December 11, 1998. The work of the second author was partially supported by CNPq, Brazil.

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