

BOUNDS FOR THE MINIMUM EIGENVALUE OF A SYMMETRIC TOEPLITZ MATRIX*

HEINRICH VOSS†

Abstract. In a recent paper Melman [12] derived upper bounds for the smallest eigenvalue of a real symmetric Toeplitz matrix in terms of the smallest roots of rational and polynomial approximations of the secular equation $f(\lambda) = 0$, the best of which being constructed by the $(1, 2)$ -Padé approximation of f . In this paper we prove that this bound is the smallest eigenvalue of the projection of the given eigenvalue problem onto a Krylov space of T_n^{-1} of dimension 3. This interpretation of the bound suggests enhanced bounds of increasing accuracy. They can be substantially improved further by exploiting symmetry properties of the principal eigenvector of T_n .

Key words. Toeplitz matrix, eigenvalue problem, symmetry.

AMS subject classifications. 65F15.

*Received November 19, 1998. Accepted for publication May 12, 1999. Communicated by L. Reichel.

† Technical University Hamburg–Harburg, Section of Mathematics, D–21071 Hamburg, Federal Republic of Germany, voss @ tu-harburg.de