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AN ANALYSIS OF LOW-RANK MODIFICATIONS OF PRECONDITIONERS FOR SADDLE POINT SYSTEMS*

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Dedicated to the memory of our mentor and friend, Gene H. Golub

Abstract. We characterize the spectral behavior of a primal Schur-complement-based block diagonal preconditioner for saddle point systems, subject to low-rank modifications. This is motivated by a desire to reduce as much as possible the computational cost of matrix-vector products with the (1,1) block, while keeping the eigenvalues of the preconditioned matrix reasonably clustered. The formulation leads to a perturbed hyperbolic quadratic eigenvalue problem. We derive interlacing results, highlighting the differences between this problem and perturbed linear eigenvalue problems. As an example, we consider primal-dual interior point methods for semidefinite programs, and express the eigenvalues of the preconditioned matrix in terms of the centering parameter.

Key words. saddle point systems, preconditioners, Schur complement, semidefinite programming

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