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TWO-LEVEL NONLINEAR ELIMINATION BASED PRECONDITIONERS FOR INEXACT NEWTON METHODS WITH APPLICATION IN SHOCKED DUCT FLOW CALCULATION*

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Abstract. The class of Newton methods is popular for solving large sparse nonlinear algebraic systems of equations arising from the discretization of partial differential equations. The method offers superlinear or quadratic convergence when the solution is sufficiently smooth and the initial guess is close to the desired solution. However, in many practical problems, the solution may exhibit some non-smoothness in part of the computational domain, due to, for example, the presence of a shock wave. In this situation, the convergence rate of Newton-type methods deteriorates considerably. In this paper, we introduce a two-level nonlinear elimination algorithm, in which we first identify a subset of equations that prevents Newton from having the fast convergence and then iteratively eliminate them from the global nonlinear system of equations. We show that such implicit nonlinear elimination restores the fast convergence for problems with local non-smoothness. As an example, we study a compressible transonic flow in a shocked duct.

Key words. nonlinear PDEs, nonlinear elimination, inexact Newton, finite difference, shock wave

AMS subject classifications. 65H10, 65N06, 65N55

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