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NEW QUADRILATERAL MIXED FINITE ELEMENTS*

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Abstract. In this paper, we introduce a new family of mixed finite element spaces of higher order ($k \geq 1$) on general quadrilateral grids. A typical element has two fewer degrees of freedom than the well-known Raviart-Thomas finite element $RT_{[k]}$, yet enjoys an optimal-order approximation for the velocity in L^2-norm. The order of approximation in the divergence norm is one less than the velocity, as is common to all other known elements, except for a recent element introduced by Arnold et al. [SIAM J. Numer. Anal., 42 (2005), pp. 2429–2451]. However, we introduce a local post-processing technique to obtain an optimal order in L^2-norm of divergence. This technique can be used to enhance the result of $RT_{[k]}$ element as well, and hence, can be easily incorporated into existing codes.

Our element has one lower order of approximation in pressure than the $RT_{[k]}$ element. However, the pressure also can be locally post-processed to produce an optimal-order approximation. The greatest advantage of our finite element lies in the fact that it has the fewest degrees of freedom among all the known quadrilateral mixed finite elements and thus, together with the post-processing techniques, provides a very efficient way of computing flow variables in mixed formulation. Numerical examples are in quite good agreement with the theory even for the case of almost degenerate quadrilateral grids.

Key words. Mixed finite element method, quadrilateral grid, optimal velocity, post-processing.

AMS subject classifications. 65N15, 65N30

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