THE STRUCTURED DISTANCE TO NEARLY NORMAL MATRICES[*]

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Dedicated to Richard S. Varga on the occasion of his 80th birthday

Abstract. In this note we examine the algebraic variety $\mathcal{I}_\Lambda$ of complex tridiagonal $n \times n$ matrices T, such that $T^*T - TT^* = \Lambda$, where Λ is a fixed real diagonal matrix. If $\Lambda = \mathbf{0}$ then $\mathcal{I}_\Lambda$ is $\mathcal{N}_\mathbf{T}$, the set of tridiagonal normal matrices. For $\Lambda \neq \mathbf{0}$, we identify the structure of the matrices in $\mathcal{I}_\Lambda$ and analyze the suitability for eigenvalue estimation using normal matrices for elements of $\mathcal{I}_\Lambda$. We also compute the Frobenius norm of elements of $\mathcal{I}_\Lambda$, describe the algebraic subvariety $\mathcal{M}_\Lambda$ consisting of elements of $\mathcal{I}_\Lambda$ with minimal Frobenius norm, and calculate the distance from a given complex tridiagonal matrix to $\mathcal{I}_\Lambda$.

Key words. nearness to normality, tridiagonal matrix, Kreĭn spaces, eigenvalue estimation, Geršgorin type sets

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