

**LAURENT POLYNOMIAL PERTURBATIONS OF LINEAR FUNCTIONALS.
AN INVERSE PROBLEM.***

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Dedicated to Richard S. Varga, on the occasion of his 80th birthday.

Abstract. Given a linear functional \mathcal{L} in the linear space \mathbb{P} of polynomials with complex coefficients, we analyze those linear functionals $\tilde{\mathcal{L}}$ such that, for a fixed $\alpha \in \mathbb{C}$, $\langle \tilde{\mathcal{L}}, (z + z^{-1} - (\alpha + \bar{\alpha}))p \rangle = \langle \mathcal{L}, p \rangle$ for every $p \in \mathbb{P}$. We obtain the relation between the corresponding Carathéodory functions in such a way that a linear spectral transform appears. If \mathcal{L} is a positive definite linear functional, the necessary and sufficient conditions in order for $\tilde{\mathcal{L}}$ to be a quasi-definite linear functional are given. The relation between the corresponding sequences of monic orthogonal polynomials is presented.

Key words. Orthogonal polynomials, linear functionals, Laurent polynomials, linear spectral transformations.

AMS subject classifications. 42C05.

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