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BOUNDARY CONDITIONS IN APPROXIMATE COMMUTATOR PRECONDITIONERS FOR THE NAVIER-STOKES EQUATIONS*

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Abstract. Boundary conditions are analyzed for a class of preconditioners used for the incompressible Navier-Stokes equations. We consider pressure convection-diffusion preconditioners [SIAM J. Sci. Comput., 24 (2002), pp. 237–256] and [J. Comput. Appl. Math., 128 (2001), pp. 261–279] as well as least-square commutator methods [SIAM J. Sci. Comput., 30 (2007), pp. 290–311] and [SIAM J. Sci. Comput., 27 (2006), pp. 1651–1668], both of which rely on commutators of certain differential operators. The effectiveness of these methods has been demonstrated in various studies, but both methods also have some deficiencies. For example, the pressure convection-diffusion preconditioner requires the construction of a Laplace and a convection–diffusion operator, together with some choices of boundary conditions. These boundary conditions are not well understood, and a poor choice can critically affect performance. This paper looks closely at properties of commutators near domain boundaries. We show that it is sometimes possible to choose boundary conditions to force the commutators of interest to be zero at boundaries, and this leads to a new strategy for choosing boundary conditions for the purpose of specifying preconditioning operators. With the new preconditioners, Krylov subspace methods display noticeably improved performance for solving the Navier-Stokes equations; in particular, mesh-independent convergence rates are observed for some problems for which previous versions of the methods did not exhibit this behavior.

Key words. boundary conditions, commutators, preconditioners, Navier-Stokes equations

AMS subject classifications. Primary: 65F10, 65N30, 76D05; Secondary: 15A06, 35Q30

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