STRUCTURAL AND RECURRENCE RELATIONS FOR HYPERGEOMETRIC-TYPE FUNCTIONS BY NIKIFOROV-UVAROV METHOD[*]

J. L. CARDOSO[†], C. M. FERNANDES[†], AND R. ÁLVAREZ-NODARSE[‡]

Abstract. The functions of hypergeometric-type are the solutions $y = y_\nu(z)$ of the differential equation $\sigma(z)y'' + \tau(z)y' + \lambda y = 0$, where σ and τ are polynomials of degrees not higher than 2 and 1, respectively, and λ is a constant. Here we consider a class of functions of hypergeometric type: those that satisfy the condition $\lambda + \nu\tau' + \frac{1}{2}\nu(\nu-1)\sigma'' = 0$, where ν is an arbitrary complex (fixed) number. We also assume that the coefficients of the polynomials σ and τ do not depend on ν. To this class of functions belong Gauss, Kummer, and Hermite functions, and also the classical orthogonal polynomials. In this work, using the constructive approach introduced by Nikiforov and Uvarov, several structural properties of the hypergeometric-type functions $y = y_\nu(z)$ are obtained. Applications to hypergeometric functions and classical orthogonal polynomials are also given.

Key words. hypergeometric-type functions, recurrence relations, classical orthogonal polynomials

AMS subject classifications. 33C45, 33C05, 33C15

[†]Departamento de Matemática, Universidade de Trás-os-Montes e Alto Douro, Apartado 202, 5001 - 911 Vila Real, Portugal (jluis@utad.pt,fernandes.cibele@gmail.com).

[‡]Departamento de Análisis Matemático, Facultad de Matemática, Universidad de Sevilla, Apdo. Postal 1160, Sevilla, E-41080, Sevilla, Spain (ran@us.es).