Electronic Transactions on Numerical Analysis.
Volume 35, pp. 118-128, 2009.

ISSN 1068-9613.

ETNA
Kent State University
http://etna.math.kent.edu

MONOTONE CONVERGENCE OF THE LANCZOS APPROXIMATIONS TO MATRIX FUNCTIONS OF HERMITIAN MATRICES*

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Abstract. When A is a Hermitian matrix, the action $f(A)b$ of a matrix function $f(A)$ on a vector b can efficiently be approximated via the Lanczos method. In this note we use M-matrix theory to establish that the 2-norm of the error of the sequence of approximations is monotonically decreasing if f is a Stieltjes transform and A is positive definite. We discuss the relation of our approach to a recent, more general monotonicity result of Druskin for Laplace transforms. We also extend the class of functions to certain product type functions. This yields, for example, monotonicity when approximating $\mathrm{sign}(A)b$ with A indefinite if the Lanczos method is performed for A^2 rather than A.

Key words. matrix functions, Lanczos method, Galerkin approximation, monotone convergence, error estimates

AMS subject classifications. 6530, 65F10, 65F50

*Received June 3, 2008. Accepted for publication January 21, 2009. Published online June 24, 2009. Recommended by V. Druskin.

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