Electronic Transactions on Numerical Analysis.
Volume 34, pp. 125-135, 2009.

ISSN 1068-9613.

ETNA
Kent State University
http://etna.math.kent.edu

CHEBYSHEV SEMI-ITERATION IN PRECONDITIONING FOR PROBLEMS INCLUDING THE MASS MATRIX*

ANDY WATHEN† AND TYRONE REES†

Dedicated to Víctor Pereyra on the occasion of his 70th birthday

Abstract. It is widely believed that Krylov subspace iterative methods are better than Chebyshev semi-iterative methods. When the solution of a linear system with a symmetric and positive definite coefficient matrix is required, the Conjugate Gradient method will compute the optimal approximate solution from the appropriate Krylov subspace, that is, it will implicitly compute the optimal polynomial. Hence a semi-iterative method, which requires eigenvalue bounds and computes an explicit polynomial, must, for just a little less computational work, give an inferior result. In this manuscript, we identify a specific situation in the context of preconditioning where finite element mass matrices arise as certain blocks in a larger matrix problem when the Chebyshev semi-iterative method is the method of choice, since it has properties which make it superior to the Conjugate Gradient method. In particular, the Chebyshev method gives preconditioners which are linear operators, whereas corresponding use of conjugate gradients would be nonlinear. We give numerical results for two example problems, the Stokes problem and a PDE control problem, where such nonlinearity causes poor convergence.

Key words. Iteration, linear systems, preconditioning, finite elements, mass matrix

AMS subject classifications. 6F10, 65N20

* Received October 2, 2008. Accepted February 11, 2009. Published online August 25, 2009. Recommended by Godela Scherer.

†Oxford University Computing Laboratory, Parks Road, Oxford, UK. ({Andy.Wathen,Tyrone.Rees}@comlab.ox.ac.uk)