SIMPLE SQUARE SMOOTHING REGULARIZATION OPERATORS[*]

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Dedicated to Gérard Meurant on the occasion of his 60th birthday

Abstract. Tikhonov regularization of linear discrete ill-posed problems often is applied with a finite difference regularization operator that approximates a low-order derivative. These operators generally are represented by a banded rectangular matrix with fewer rows than columns. They therefore cannot be applied in iterative methods that are based on the Arnoldi process, which requires the regularization operator to be represented by a square matrix. This paper discusses two approaches to circumvent this difficulty: zero-padding the rectangular matrices to make them square and extending the rectangular matrix to a square circulant. We also describe how to combine these operators by weighted averaging and with orthogonal projection. Applications to Arnoldi and Lanczos bidiagonalization-based Tikhonov regularization, as well as to truncated iteration with a range-restricted minimal residual method, are presented.

Key words. ill-posed problem, regularization operator, Tikhonov regularization, truncated iteration.

AMS subject classifications. 65F10, 65F22, 65R32