

USING FGMRES TO OBTAIN BACKWARD STABILITY IN MIXED PRECISION*

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Dedicated to Gérard Meurant on the occasion of his 60th birthday

Abstract. We consider the triangular factorization of matrices in single-precision arithmetic and show how these factors can be used to obtain a backward stable solution. Our aim is to obtain double-precision accuracy even when the system is ill-conditioned. We examine the use of iterative refinement and show by example that it may not converge. We then show both theoretically and practically that the use of FGMRES will give us the result that we desire with fairly mild conditions on the matrix and the direct factorization. We perform extensive experiments on dense matrices using MATLAB and indicate how our work extends to sparse matrix factorization and solution.

Key words. FGMRES, mixed precision arithmetic, hybrid method, direct factorization, iterative methods, large sparse systems, error analysis

AMS subject classifications. 65F05, 65F10, 65F50, 65G20, 65G50

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