A NEW ITERATION FOR COMPUTING THE EIGENVALUES OF SEMISEPARABLE (PLUS DIAGONAL) MATRICES[*]

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Abstract. This paper proposes a new type of iteration for computing eigenvalues of semiseparable (plus diagonal) matrices based on a structured-rank factorization. Remarks on higher order semiseparability ranks are also made. More precisely, instead of the traditional QR iteration, a QH iteration is used. The QH factorization is characterized by a unitary matrix Q and a Hessenberg-like matrix H in which the lower triangular part is semiseparable (often called a lower semiseparable matrix). The Q factor of this factorization determines the similarity transformation of the QH method.

It is shown that this iteration is extremely useful for computing the eigenvalues of structured-rank matrices. Whereas the traditional QR method applied to semiseparable (plus diagonal) and Hessenberg-like matrices uses similarity transformations involving $2p(n-1)$ Givens transformations (where p denotes the semiseparability rank), the QH iteration only needs $p(n-1)$ Givens transformations, which is comparable to the generalized Hessenberg (symmetric band) situation having p subdiagonals. It is also shown that this method can in some sense be interpreted as an extension of the traditional QR method for Hessenberg matrices, i.e., the traditional case also fits into this framework. It is also shown that this iteration exhibits an extra type of convergence behavior compared to the traditional QR method.

The algorithm is implemented in an implicit way, based on the Givens-weight representation of the structured rank matrices. Numerical experiments show the viability of this approach. The new approach yields better complexity and more accurate results than the traditional QR method.

Key words. QH algorithm, structured rank matrices, implicit computations, eigenvalue, QR algorithm, rational QR iteration

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