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ON A MULTILEVEL KRYLOV METHOD FOR THE HELMHOLTZ EQUATION PRECONDITIONED BY SHIFTED LAPLACIAN*

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Abstract. In Erlangga and Nabben [SIAM J. Sci. Comput., 30 (2008), pp. 1572–1595], a multilevel Krylov method is proposed to solve linear systems with symmetric and nonsymmetric matrices of coefficients. This multilevel method is based on an operator which shifts some small eigenvalues to the largest eigenvalue, leading to a spectrum which is favorable for convergence acceleration of a Krylov subspace method. This shift technique involves a subspace or coarse-grid solve. The multilevel Krylov method is obtained via a recursive application of the shift operator on the coarse-grid system. This method has been applied successfully to 2D convection-diffusion problems for which a standard multigrid method fails to converge.

In this paper, we extend this multilevel Krylov method to indefinite linear systems arising from a discretization of the Helmholtz equation, preconditioned by shifted Laplacian as introduced by Erlangga, Oosterlee and Vuik [SIAM J. Sci. Comput. 27 (2006), pp. 1471–1492]. Within the Krylov iteration and the multilevel steps, for each coarse-grid solve a multigrid iteration is used to approximately invert the shifted Laplacian preconditioner. Hence, a multilevel Krylov-multigrid (MKMG) method results.

Numerical results are given for high wavenumbers and show the effectiveness of the method for solving Helmholtz problems. Not only can the convergence be made almost independent of grid size h, but also linearly dependent on the wavenumber k, with a smaller proportional constant than for the multigrid preconditioned version, presented in the aforementioned paper.

Key words. multilevel Krylov method, GMRES, multigrid, Helmholtz equation, shifted-Laplace preconditioner.

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