

## ALGORITHMS FOR THE MATRIX SECTOR FUNCTION\*

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**Abstract.** In this paper we consider algorithms for the matrix sector function, which is a generalization of the matrix sign function. We develop algorithms for computing the matrix sector function based on the (real) Schur decompositions, with and without reordering and the Parlett recurrence. We prove some results on the convergence regions for the specialized versions of Newton's and Halley's methods applied to the matrix sector function, using recent results of Iannazzo for the principal matrix  $p$ th root. Numerical experiments comparing the properties of algorithms developed in this paper illustrate the differences in the behaviour of the algorithms. We consider the conditioning of the matrix sector function and the stability of Newton's and Halley's methods. We also prove a characterization of the Fréchet derivative of the matrix sector function, which is a generalization of the result of Kenney and Laub for the Fréchet derivative of the matrix sign function, and we provide a way of computing it by Newton's iteration.

**Key words.** matrix sector function, matrix sign function, matrix  $p$ th root, Schur algorithm, Parlett recurrence, Newton's method, Halley's method, stability, conditioning, Fréchet derivative.

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