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CROSS-GRAMIAN BASED MODEL REDUCTION FOR DATA-SPARSE SYSTEMS*

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Abstract. Model order reduction (MOR) is common in simulation, control and optimization of complex dynamical systems arising in modeling of physical processes, and in the spatial discretization of parabolic partial differential equations in two or more dimensions. Typically, after a semi-discretization of the differential operator by the finite or boundary element method, we have a large state-space dimension n. In order to accelerate the simulation time or to facilitate the control design, it is often desirable to employ an approximate reduced-order system of order r, with $r \ll n$, instead of the original large-scale system. We show how to compute a reduced-order system with a balancing-related model reduction method. The method is based on the computation of the cross-Gramian $\mathcal{X}$, which is the solution of a Sylvester equation. As standard algorithms for the solution of Sylvester equations are of limited use for large-scale (possibly dense) systems, we investigate approaches based on the iterative sign function method, using data-sparse matrix approximations (the hierarchical matrix format) and an approximate arithmetic. Furthermore, we use a modified iteration scheme for computing low-rank factors of the solution $\mathcal{X}$. The projection matrices for MOR are computed from the dominant invariant subspace of $\mathcal{X}$. We propose an efficient algorithm for the direct calculation of these projectors from the low-rank factors of $\mathcal{X}$. Numerical experiments demonstrate the performance of the new approach.

Key words. Model reduction, balanced truncation, cross-Gramian, hierarchical matrices, sign function method.

AMS subject classifications. 93B11, 93B40, 93C20, 37M05.

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