

## NOISE PROPAGATION IN REGULARIZING ITERATIONS FOR IMAGE DEBLURRING\*

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**Abstract.** We use the two-dimensional discrete cosine transform to study how the noise from the data enters the reconstructed images computed by regularizing iterations, that is, Krylov subspace methods applied to discrete ill-posed problems. The regularization in these methods is obtained via the projection onto the associated Krylov subspace. We focus on CGLS/LSQR, GMRES, and RRGMR, as well as MINRES and MR-II in the symmetric case. Our analysis shows that the noise enters primarily in the form of band-pass filtered white noise, which appears as “freckles” in the reconstructions, and these artifacts are present in both the signal and the noise components of the solutions. We also show why GMRES and MINRES are not suited for image deblurring.

**Key words.** Image deblurring, regularizing iterations, Krylov subspaces, CGLS, LSQR, GMRES, MINRES, RRGMR, MR-II.

**AMS subject classifications.** 65F22, 65F10.

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\*Received November 28, 2007. Accepted August 18, 2008. Published online on March 5, 2008. Recommended by Zdeněk Strakoš. This work was carried out as part of the project CSI: Computational Science in Imaging, supported by grant no. 274-07-0065 from the Danish Agency for Science Technology and Innovation.

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