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APPROXIMATION OF THE SCATTERING AMPLITUDE AND LINEAR SYSTEMS*

GENE H. GOLUB†, MARTIN STOLL‡, AND ANDY WATHEN‡

Abstract. The simultaneous solution of $Ax = b$ and $A^T y = g$, where A is a non-singular matrix, is required in a number of situations. Darmofal and Lu have proposed a method based on the Quasi-Minimal Residual algorithm (QMR). We will introduce a technique for the same purpose based on the LSQR method and show how its performance can be improved when using the generalized LSQR method. We further show how preconditioners can be introduced to enhance the speed of convergence and discuss different preconditioners that can be used. The scattering amplitude $g^T x$, a widely used quantity in signal processing for example, has a close connection to the above problem since x represents the solution of the forward problem and g is the right-hand side of the adjoint system. We show how this quantity can be efficiently approximated using Gauss quadrature and introduce a block-Lanczos process that approximates the scattering amplitude, and which can also be used with preconditioning.

Key words. Linear systems, Krylov subspaces, Gauss quadrature, adjoint systems.

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†Department of Computer Science, Stanford University, Stanford, CA94305-9025, USA.

‡Oxford University Computing Laboratory, Wolfson Building, Parks Road, Oxford, OX1 3QD, United Kingdom ({martin.stoll,andy.wathen}@comlab.ox.ac.uk).