

MAJORIZATION BOUNDS FOR RITZ VALUES OF HERMITIAN MATRICES*

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Abstract. Given an approximate invariant subspace we discuss the effectiveness of majorization bounds for assessing the accuracy of the resulting Rayleigh-Ritz approximations to eigenvalues of Hermitian matrices. We derive a slightly stronger result than previously for the approximation of k extreme eigenvalues, and examine some advantages of these majorization bounds compared with classical bounds. From our results we conclude that the majorization approach appears to be advantageous, and that there is probably much more work to be carried out in this direction.

Key words. Hermitian matrices, angles between subspaces, majorization, Lidskii's eigenvalue theorem, perturbation bounds, Ritz values, Rayleigh-Ritz method, invariant subspace.

AMS subject classifications. 15A18, 15A42, 15A57.

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