

ON THE PARAMETER SELECTION PROBLEM IN THE NEWTON-ADI ITERATION FOR LARGE-SCALE RICCATI EQUATIONS*

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Abstract. The numerical treatment of linear-quadratic regulator (LQR) problems for parabolic partial differential equations (PDEs) on infinite-time horizons requires the solution of large-scale algebraic Riccati equations (AREs). The Newton-ADI iteration is an efficient numerical method for this task. It includes the solution of a Lyapunov equation by the alternating direction implicit (ADI) algorithm at each iteration step. Here, we study the selection of shift parameters for the ADI method. This leads to a rational min-max problem which has been considered by many authors. Since knowledge about the exact shape of the complex spectrum is crucial for computing the optimal solution, this is often infeasible for the large-scale systems arising from finite element discretization of PDEs. Therefore, several methods for computing suboptimal parameters are discussed and compared on numerical examples.

Key words. algebraic Riccati equation, Newton-ADI, shift parameters, Lyapunov equation, rational min-max problem, Zolotarev problem

AMS subject classifications. 15A24, 30E10, 65B99

*Received October 31, 2006. Accepted for publication March 7, 2008. Published online on June 26, 2008. Recommended by A. Frommer. This work was supported by the DFG project “Numerische Lösung von Optimalsteuerungsproblemen für instationäre Diffusions-Konvektions- und Diffusions-Reaktionsgleichungen”, grant BE3715/1-1 and DAAD program “Acciones Integradas Hispano-Alemanas”, grant D/05/25675.

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