

A GENERALIZATION OF THE STEEPEST DESCENT METHOD FOR MATRIX FUNCTIONS*

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In memory of Gene Golub

Abstract. We consider the special case of the restarted Arnoldi method for approximating the product of a function of a Hermitian matrix with a vector which results when the restart length is set to one. When applied to the solution of a linear system of equations, this approach coincides with the method of steepest descent. We show that the method is equivalent to an interpolation process in which the node sequence has at most two points of accumulation. This knowledge is used to quantify the asymptotic convergence rate.

Key words. Matrix function, Krylov subspace approximation, restarted Krylov subspace method, restarted Arnoldi/Lanczos method, linear system of equations, steepest descent, polynomial interpolation

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