

A WEIGHTED-GCV METHOD FOR LANCZOS-HYBRID REGULARIZATION*

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In memory of Gene Golub

Abstract. Lanczos-hybrid regularization methods have been proposed as effective approaches for solving large-scale ill-posed inverse problems. Lanczos methods restrict the solution to lie in a Krylov subspace, but they are hindered by semi-convergence behavior, in that the quality of the solution first increases and then decreases. Hybrid methods apply a standard regularization technique, such as Tikhonov regularization, to the projected problem at each iteration. Thus, regularization in hybrid methods is achieved both by Krylov filtering and by appropriate choice of a regularization parameter at each iteration. In this paper we describe a weighted generalized cross validation (W-GCV) method for choosing the parameter. Using this method we demonstrate that the semi-convergence behavior of the Lanczos method can be overcome, making the solution less sensitive to the number of iterations.

Key words. generalized cross validation, ill-posed problems, iterative methods, Lanczos bidiagonalization, LSQR, regularization, Tikhonov

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