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**ETNA**
**Kent State University**
**etna@mcs.kent.edu**

# THE DIFFERENCE EQUATION RELATED TO THE PROBLEM OF THE HYDROGEN ATOM IN A STRONG MAGNETIC FIELD*

MARKO ROBNIK† AND VALERY G. ROMANOVSKI†

**Abstract.** We study the Schrödinger equation for the hydrogen atom in an arbitrarily strong magnetic field in two dimensions, which is an integrable and separable system. The energy spectrum is very interesting as it has infinitely many accumulation points located at the values of the Landau energy levels of a free electron in the uniform magnetic field. In the polar coordinates the canonical (not kinetic!) angular momentum has a precise eigenvalue and we have the one dimensional radial Schrödinger equation which is an ordinary second order differential equation whose analytic exact solution is unknown. The problem is reduced to a linear three-term recurrence difference equation whose solution is unknown. We describe the qualitative properties of the energy spectrum and propose a semi-analytic method to numerically calculate the eigenenergies.

**Key words.** hydrogen atom, strong magnetic field

**AMS subject classifications.** 81V45, 81Q10

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†CAMTP–Center for Applied Mathematics and Theoretical Physics, University of Maribor, Maribor, Krekova 2, SI-2000, Slovenia (robnik, valery.romanovsky@uni-mb.si).