

LEFT-DEFINITE VARIATIONS OF THE CLASSICAL FOURIER EXPANSION THEOREM*

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Abstract. In a recent paper, Littlejohn and Wellman developed a general left-definite theory for arbitrary self-adjoint operators in a Hilbert space that are bounded below by a positive constant. We apply this theory and construct the sequences of left-definite Hilbert spaces $\{H_n\}_{n \in \mathbb{N}}$ and left-definite self-adjoint operators $\{A_n\}_{n \in \mathbb{N}}$ associated with the classical, regular self-adjoint boundary value problem consisting of the Fourier equation with periodic boundary conditions. As a particular consequence of our analysis, we obtain a Fourier expansion theorem in each left-definite space H_n as well as an explicit representation of the domain of $A^{n/2}$ for each positive integer n .

Key words. self-adjoint operator, Hilbert space, left-definite Hilbert space, left-definite operator, regular self-adjoint boundary value problem, Fourier series

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