

ORTHOGONAL POLYNOMIALS AND RECURRENCE EQUATIONS, OPERATOR EQUATIONS AND FACTORIZATION*

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Abstract. This article surveys the classical orthogonal polynomial systems of the Hahn class, which are solutions of second-order differential, difference or q -difference equations.

Orthogonal families satisfy three-term recurrence equations. Example applications of an algorithm to determine whether a three-term recurrence equation has solutions in the Hahn class—implemented in the computer algebra system *Maple*—are given.

Modifications of these families, in particular associated orthogonal systems, satisfy fourth-order operator equations. A factorization of these equations leads to a solution basis.

Key words. orthogonal polynomials, Hahn class, differential equations, difference equations, q -difference equations, hypergeometric functions, factorization of operator polynomials, computer algebra, Maple

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