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COMPUTING QUATERNIONIC ROOTS BY NEWTON'S METHOD*

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Abstract. Newton's method for finding zeros is formally adapted to finding roots of Hamilton's quaternions. Since a derivative in the sense of complex analysis does not exist for quaternion valued functions we compare the resulting formulas with the more classical formulas obtained by using the Jacobian matrix and the Gâteaux derivative. The latter case includes also the so-called damped Newton form. We investigate the convergence behavior and show that under one simple condition all cases introduced, produce the same iteration sequence and have thus the same convergence behavior, namely that of locally quadratic convergence. By introducing an analogue of Taylor's formula for $x^n,\ n \in \mathbb{Z}$, we can show the local, quadratic convergence independently of the general theory. It will also be shown that the application of damping proves to be very useful. By applying Newton iterations backwards we detect all points for which the iteration (after a finite number of steps) must terminate. These points form a nice pattern. There are explicit formulas for roots of quaternions and also numerical examples.

Key words. Roots of quaternions, Newton's method applied to finding roots of quaternions.

AMS subject classifications. 11R52, 12E15, 30G35, 65D15

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