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THEORY AND NUMERICS FOR MULTI-TERM PERIODIC DELAY DIFFERENTIAL EQUATIONS: SMALL SOLUTIONS AND THEIR DETECTION*

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Abstract. In this paper we consider scalar linear periodic delay differential equations of the form

$$x'(t) = \sum_{j=0}^{m} b_j(t)x(t-jw), x(t) = \phi(t) \text{ for } t \in [0, mw),\ t \geq mw \quad (\ddagger)$$

where b_j, $j = 0, ..., m$ are continuous periodic functions with period w. We summarise a theoretical treatment that analyses whether the equation has small solutions. We consider discrete equations that arise when a numerical method with fixed step-size is applied to approximate the solution to (‡) and we develop a corresponding theory. Our results show that small solutions can be detected reliably by the numerical scheme. We conclude with some numerical examples.

Key words. delay differential equations, small solutions, super-exponential solutions, numerical methods

AMS subject classifications. 34K28, 65P99, 37N30

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