Electronic Transactions on Numerical Analysis.
Volume 26, pp. 453-473, 2007.

ISSN 1068-9613.

ETNA
Kent State University
etna@mcs.kent.edu

ON WORST-CASE GMRES, IDEAL GMRES, AND THE POLYNOMIAL NUMERICAL HULL OF A JORDAN BLOCK*

PETR TICHÝ*, JÖRG LIESEN†, AND VANCE FABER‡

Abstract. When solving a linear algebraic system $Ax = b$ with GMRES, the relative residual norm at each step is bounded from above by the so-called ideal GMRES approximation. This worst-case bound is sharp (i.e. it is attainable by the relative GMRES residual norm) in case of a normal matrix A, but it need not characterize the worst-case GMRES behavior if A is nonnormal. Characterizing the tightness of this bound for nonnormal matrices A represents an important and largely open problem in the convergence analysis of Krylov subspace methods. In this paper we address this problem in case A is a single Jordan block. We study the relation between ideal and worst-case GMRES as well as the problem of estimating the ideal GMRES approximation. Furthermore, we prove new results about the radii of the polynomial numerical hulls of Jordan blocks. Using these, we discuss the closeness of the lower bound on the ideal GMRES approximation that is derived from the radius of the polynomial numerical hull.

Key words. GMRES convergence, ideal GMRES, polynomial numerical hull, Jordan block.

AMS subject classifications. 65F10, 65F35, 49K35.

---

* Received March 6, 2007. Accepted for publication September 17, 2007. Recommended by M. Gutknecht. The work of the first author was supported by the Emmy Noether-Programm of the Deutsche Forschungsgemeinschaft and by the Czech National Program of Research "Information Society" under project 1ET400300415. The work of the second author was supported by the Emmy Noether-Programm of the Deutsche Forschungsgemeinschaft.

* Institute of Computer Science, Academy of Sciences of the Czech Republic, Pod vodárenskou věží 2, 18207 Prague, Czech Republic (tichy@cs.cas.cz).

† Institute of Mathematics, Technical University of Berlin, Straße des 17. Juni 136, 10623 Berlin, Germany (liesen@math.tu-berlin.de).

‡ BD Biosciences - Bioimaging Systems (vance_faber@bd.com).