

## STABILITY PROPERTIES OF DIFFERENTIAL-ALGEBRAIC EQUATIONS AND SPIN-STABILIZED DISCRETIZATIONS\*

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**Abstract.** Classical stability properties of solutions that are well-known for ordinary differential equations (ODEs) are generalized to differential-algebraic equations (DAEs). A new test equation is derived for the analysis of numerical methods applied to DAEs with respect to the stability of the numerical approximations. Moreover, a stabilization technique is developed to improve the stability of classical DAE integration methods. The stability regions for these stabilized discretization methods are determined and it is shown that they much better reproduce the stability properties known for the ODE case than in the unstabilized form. Movies that depict the stability regions for several methods are included for interactive use.

**Key words.** nonlinear differential-algebraic equations, stability, asymptotic stability, Lyapunov stability, spin-stabilized discretization, test equation, strangeness index

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