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ITERATIVE METHODS FOR SOLVING THE DUAL FORMULATION ARISING FROM IMAGE RESTORATION*

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Abstract. Many variational models for image denoising restoration are formulated in primal variables that are directly linked to the solution to be restored. If the total variation related semi-norm is used in the models, one consequence is that extra regularization is needed to remedy the highly non-smooth and oscillatory coefficients for effective numerical solution. The dual formulation was often used to study theoretical properties of a primal formulation. However as a model, this formulation also offers some advantages over the primal formulation in dealing with the above mentioned oscillation and non-smoothness. This paper presents some preliminary work on speeding up the Chambolle method [J. Math. Imaging Vision, 20 (2004), pp. 89–97] for solving the dual formulation. Following a convergence rate analysis of this method, we first show why the nonlinear multigrid method encounters some difficulties in achieving convergence. Then we propose a modified smoother for the multigrid method to enable it to achieve convergence in solving a regularized Chambolle formulation. Finally, we propose a linearized primal-dual iterative method as an alternative stand-alone approach to solve the dual formulation without regularization. Numerical results are presented to show that the proposed methods are much faster than the Chambolle method.

Key words. image restoration, nonlinear partial differential equations, singularity, nonlinear iterations, Fourier analysis, multigrid method

AMS subject classifications. 68U10, 65F10, 65K10

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