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JOINT DOMAIN-DECOMPOSITION $\mathcal{H}$-LU PRECONDITIONERS FOR SADDLE POINT PROBLEMS*

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Abstract. For saddle point problems in fluid dynamics, several popular preconditioners exploit the block structure of the problem to construct block triangular preconditioners. The performance of such preconditioners depends on whether fast, approximate solvers for the linear systems on the block diagonal (representing convection-diffusion problems) as well as for the Schur complement (in the pressure variables) are available. In this paper, we will introduce a completely different approach in which we ignore this given block structure. We will instead compute an approximate LU-factorization of the complete system matrix using hierarchical matrix techniques. In particular, we will use domain-decomposition clustering with an additional local pivoting strategy to order the complete index set. As a result, we obtain an $\mathcal{H}$-matrix structure in which an $\mathcal{H}$-LU factorization is computed more efficiently and with higher accuracy than for the corresponding block structure based clustering. $\mathcal{H}$-LU preconditioners resulting from the block and joint approaches will be discussed and compared through numerical results.

Key words. hierarchical matrices, data-sparse approximation, Oseen equations, preconditioning, factorization

AMS subject classifications. 65F05, 65F30, 65F50

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