Electronic Transactions on Numerical Analysis.
Volume 26, pp. 243-269, 2007.

ISSN 1068-9613.

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SOLVING LINEAR SYSTEMS WITH A LEVINSON-LIKE SOLVER*

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Abstract. In this paper we will present a general framework for solving linear systems of equations. The solver is based on the Levinson-idea for solving Toeplitz systems of equations. We will consider a general class of matrices, defined as the class of simple (p_1, p_2)-Levinson conform matrices. This class incorporates, for instance, semiseparable, band, companion, arrowhead and many other matrices. For this class, we will derive a solver of complexity $O(p_1p_2n)$. The system solver is written inductively, and uses in every step k, the solution of a so-called kth order Yule-Walker-like equation. The algorithm obtained first has complexity $O(p_1p_2n^2)$. Based, however on the specific structure of the simple (p_1, p_2)-Levinson conform matrices, we will be able to further reduce the complexity of the presented method, and get an order $O(p_1p_2n)$ algorithm.

Different examples of matrices are given for this algorithm. Examples are presented for: general dense matrices, upper triangular matrices, higher order generator semiseparable matrices, quasiseparable matrices, Givens-vector representable semiseparable matrices, band matrices, companion matrices, confederate matrices, arrowhead matrices, fellow matrices and many more.

Finally, the relation between this method and an upper triangular factorization of the original matrix is given and also details concerning possible look ahead methods are presented.

Key words. Levinson, Yule-Walker, look-ahead, system solving, Levinson conform matrices

AMS subject classifications. 65F05

*Received August 24, 2005. Accepted for publication December 23, 2006. Recommended by P. Van Dooren. The research was partially supported by the Research Council K.U.Leuven, projects OT/00/16 (SLAP: Structured Linear Algebra Package), OT/05/40 (Large rank structured matrix computations), by the Fund for Scientific Research–Flanders (Belgium), projects G.0078.01 (SMA: Structured Matrices and their Applications), G.0176.02 (ANCILA: Asymptotic aNalysis of the Convergence behavior of Iterative methods in numerical Linear Algebra), G.0184.02 (CORFU: Constructive study of Orthogonal Functions) and G.0455.0 (RHPH: Riemann-Hilbert problems, random matrices and Padé-Hermite approximation), and by the Belgian Programme on Interuniversity Poles of Attraction, initiated by the Belgian State, Prime Minister's Office for Science, Technology and Culture, project IUAPV-22 (Dynamical Systems and Control: Computation, Identification & Modelling). The research of the second author was partially supported by MIUR, grant number 2004015437, by the short term mobility program, Consiglio Nazionale delle Ricerche and by VII Programma Esecutivo di Collaborazione Scientifica Italia–Comunità Francese del Belgio, 2005–2006. The scientific responsibility rests with the authors.

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