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THE PROPERTIES, INEQUALITIES AND NUMERICAL APPROXIMATION OF MODIFIED BESSEL FUNCTIONS*

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Dedicated to Ed Saff on the occasion of his 60th birthday

Abstract. Some new properties of kernels of modified Kontorovitch–Lebedev integral transforms — modified Bessel functions of the second kind with complex order $K_{\frac{1}{2}+i\beta}(x)$ are presented. Inequalities giving estimations for these functions with argument x and parameter β are obtained. The polynomial approximations of these functions as a solutions of linear differential equations with polynomial coefficients and their systems are proposed.

Key words. Chebyshev polynomials, modified Bessel functions, Lanczos Tau method, Kontorovich-Lebedev integral transforms

AMS subject classifications. 33C10, 33F05, 65D20

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